SPRING MEETING

MAY 11-13, 2020 • ONLINE EVENT

How to Allocate Capital if You Really Must

John A. Major
Guy Carpenter

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Convex Risk LLC

Polling Question 1

What best describes your employment?

(a) insurer/reinsurer
(b) consultant/broker
(c) academic/educator
(d) government
(e) non-insurance financial institution
(f) other

Polling Question 2

Do you have first-hand knowledge of the capital or capital cost allocation procedure(s) at your company or elsewhere?

(a) Yes

(b) No

Polling Question 3

If you answered YES to the previous question, please answer this one. How satisfied are you with those allocation procedures?

(a) Satisfied

(b) Mixed

(c) Dissatisfied

GUY CARPENTER

SPECTRAL RISK MEASURES AND PRICING INSURANCE RISK: PART 1

CAS SPRING MEETING

MAY 13, 2020

John A. Major, ASA, MAAA
Senior Vice President

New York

MARSH & McLENNAN COMPANIES

About this work…

- We are **NOT**
  - Addressing theoreticians
  - Explaining market prices
    - "What is the correct risk premium?"

About this work…

- We are **NOT**
  - Addressing theoreticians
  - Explaining market prices
    - "What is the correct risk premium?"

- We **ARE**
  - Addressing working actuaries
  - Presenting a framework for portfolio management
    - "Which business segments are over- or under-priced?"

Assumptions / strategy

- External rule determines assets required
- Law-invariant rule prices the portfolio
- Business segments get equal priority in default
- Ignore expenses, investment income, and debt

- Relentlessly layer-focused

The reasons we are often tempted to allocate capital

- Pricing
- Underwriting
- Line of business performance assessment
- Reinsurance decision making
- Growth planning
- Capital planning
- Etc.

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- Why?
  – Because we think that allocated capital is relevant to these questions

What we really want to do

- Allocate the ***cost*** of capital
  - Aggregate (pooled) cost usually an input
  - But how to allocate?

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  - Allocate capital,
  - then multiply by (one, unique) hurdle rate

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- How we typically do it ("Industry Standard Approach"):
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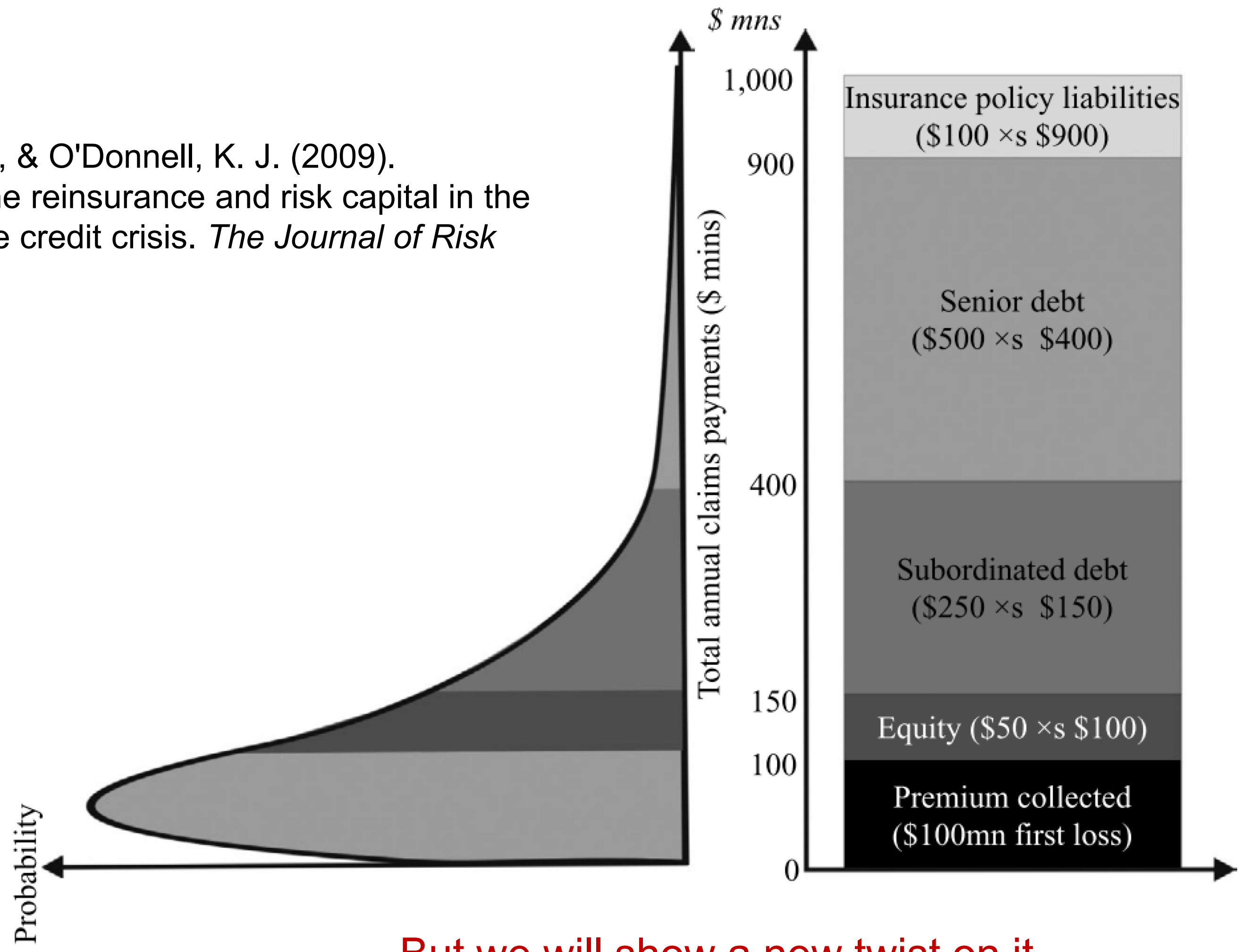
- Fallacy:
  - "Capital is fungible, so every unit requires the same return"
  - ***FALSE***

This is not a new insight

Culp, C. L., & O'Donnell, K. J. (2009). Catastrophe reinsurance and risk capital in the wake of the credit crisis. *The Journal of Risk Finance*.

Figure 3



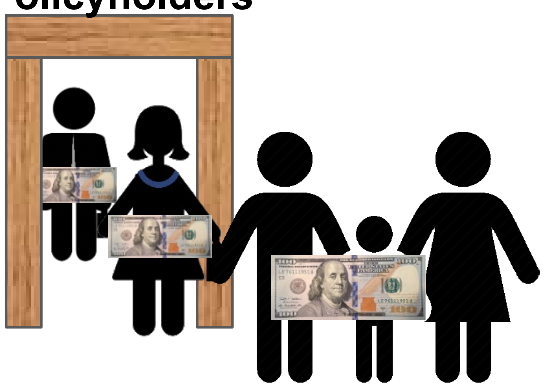
But we will show a new twist on it

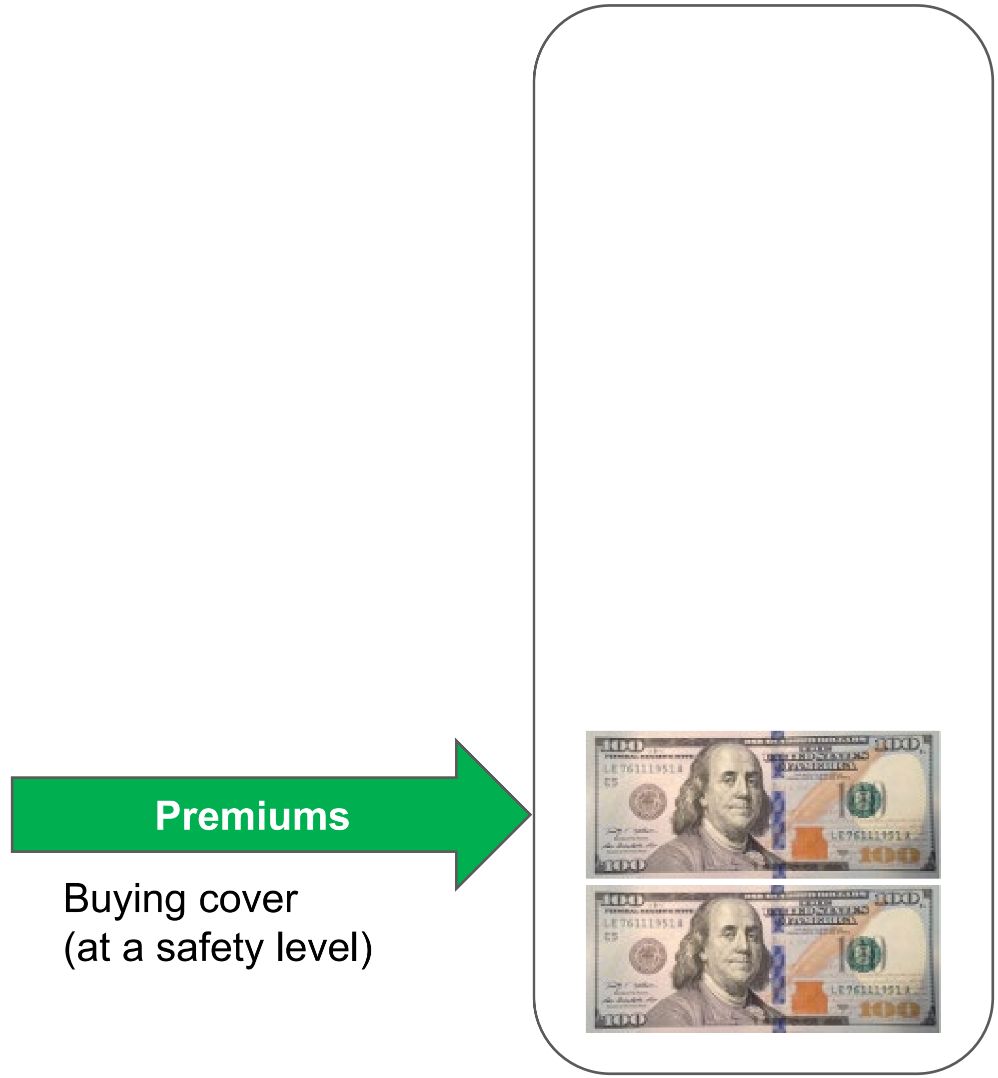
How we usually think about operations: (1) funding assets

NOTE:

- *Premium is net of expenses*
- *Claims are net of expenses*
- *Risk-free rate is 0*

Policyholders





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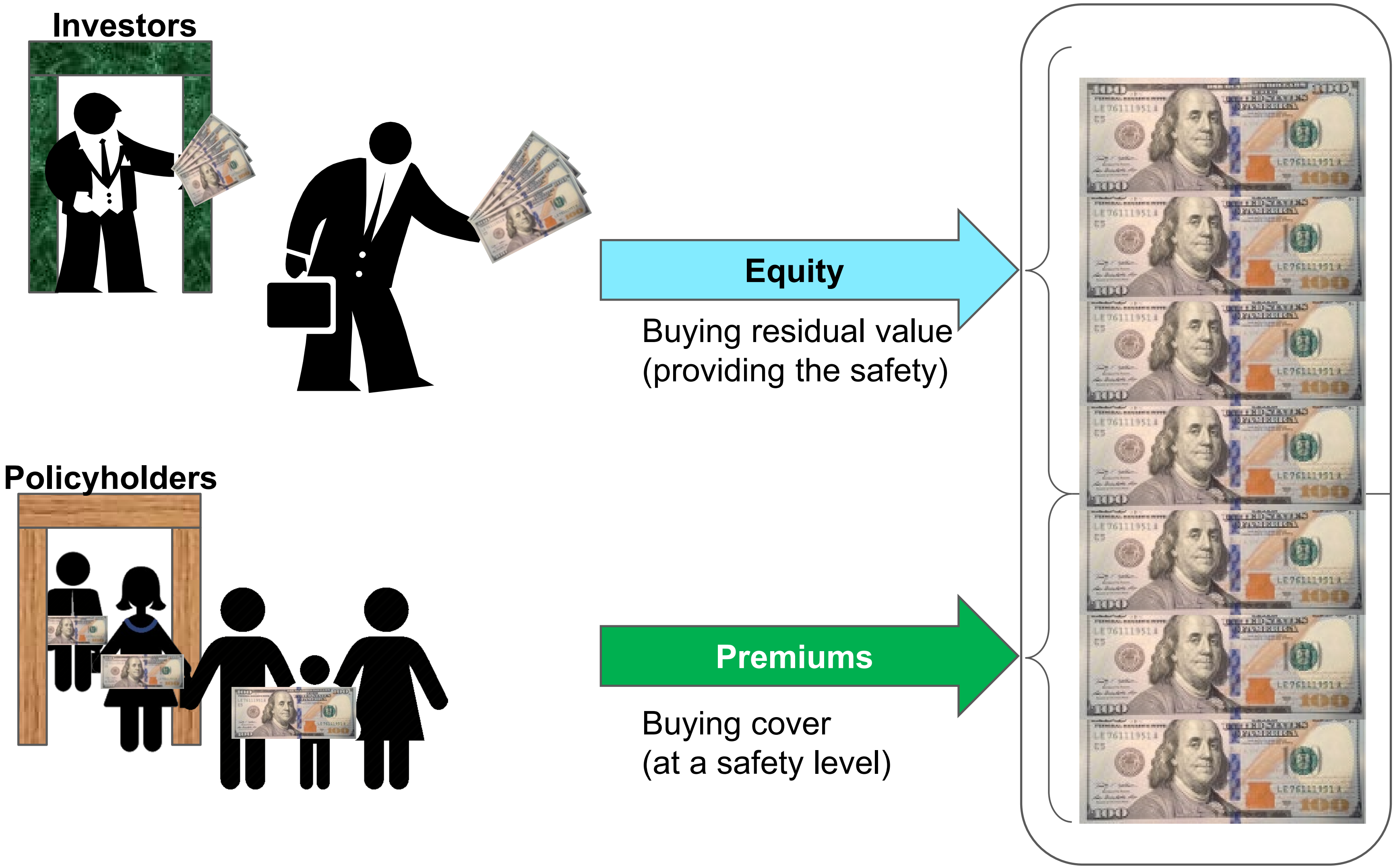
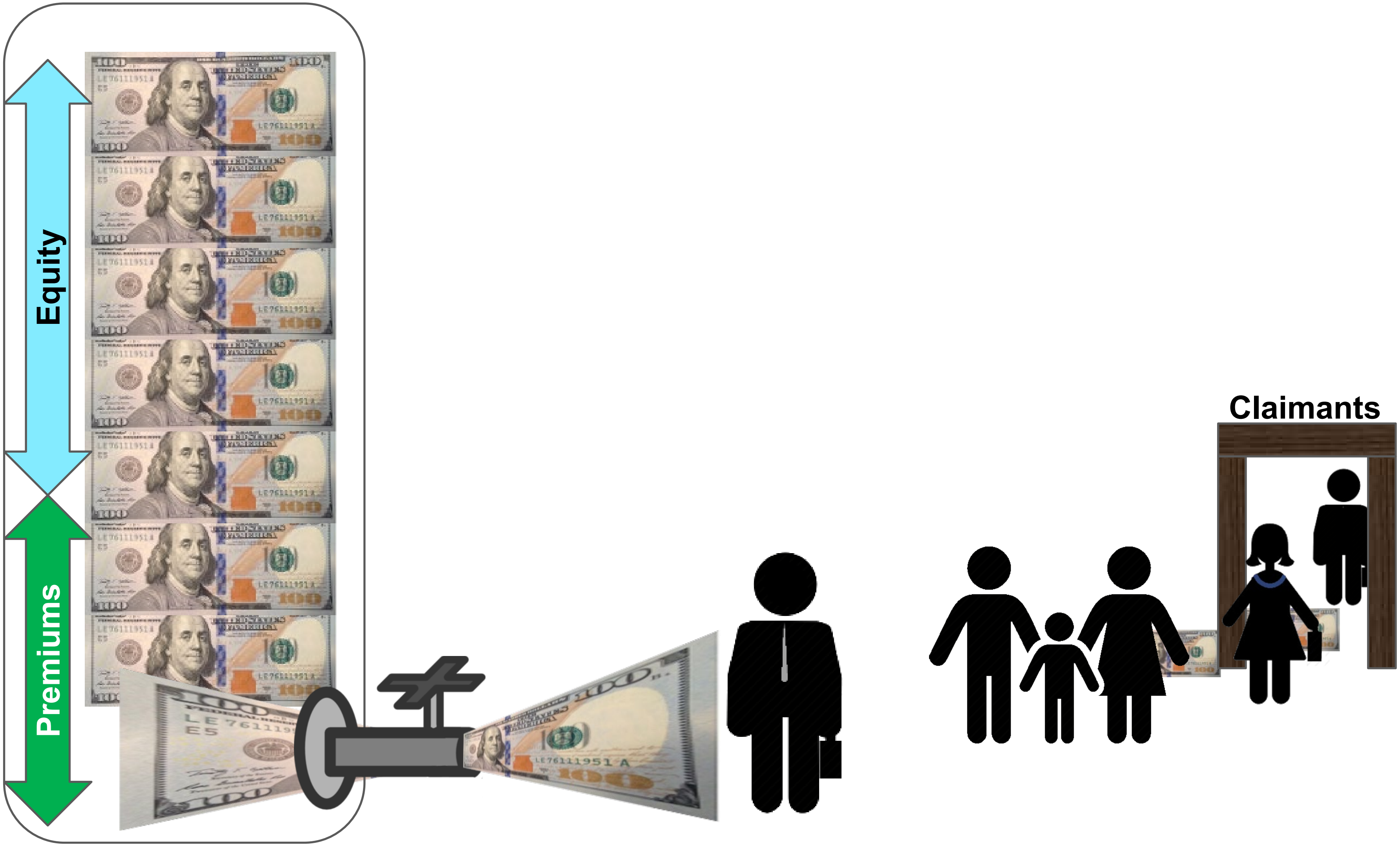
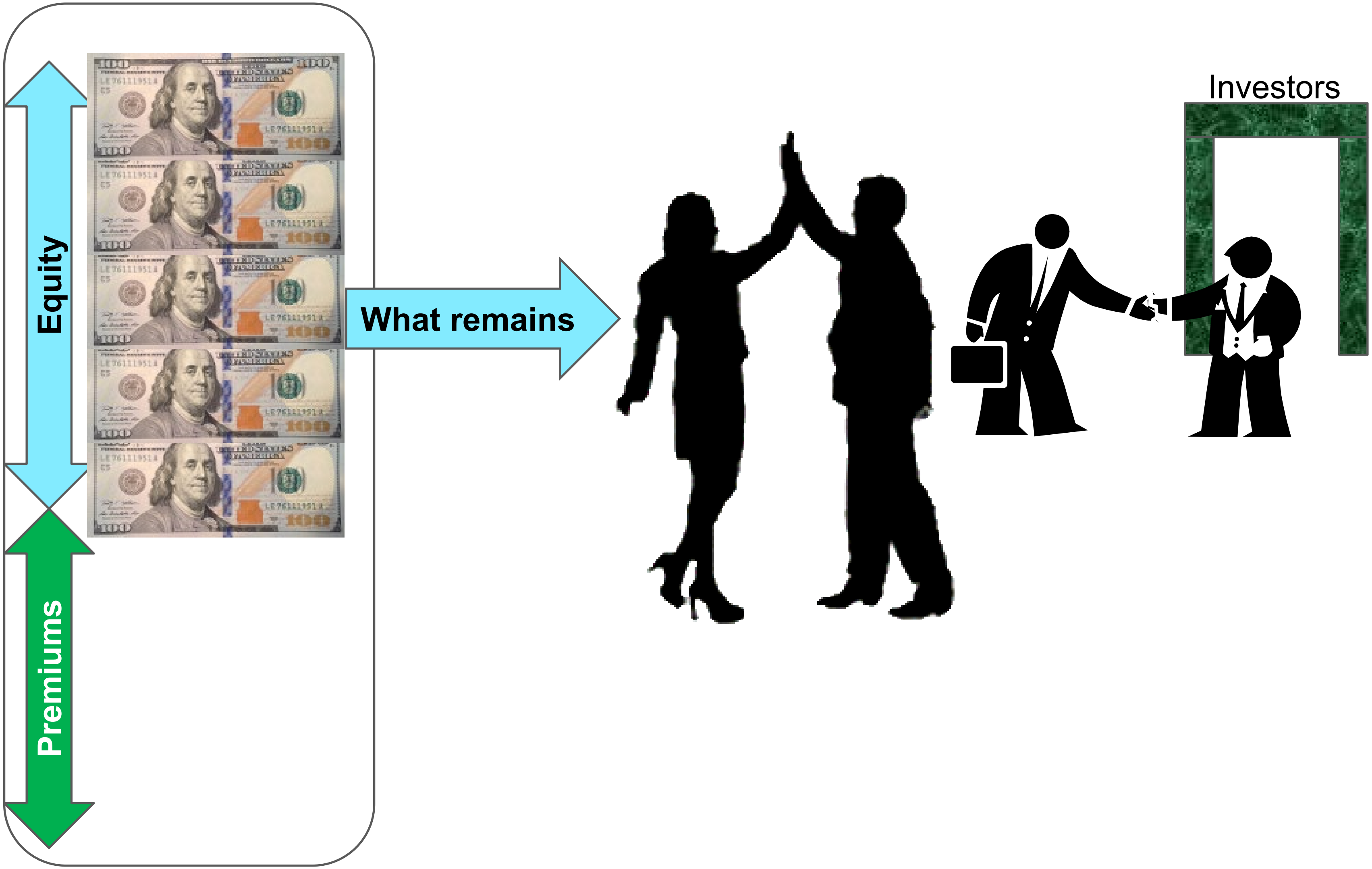


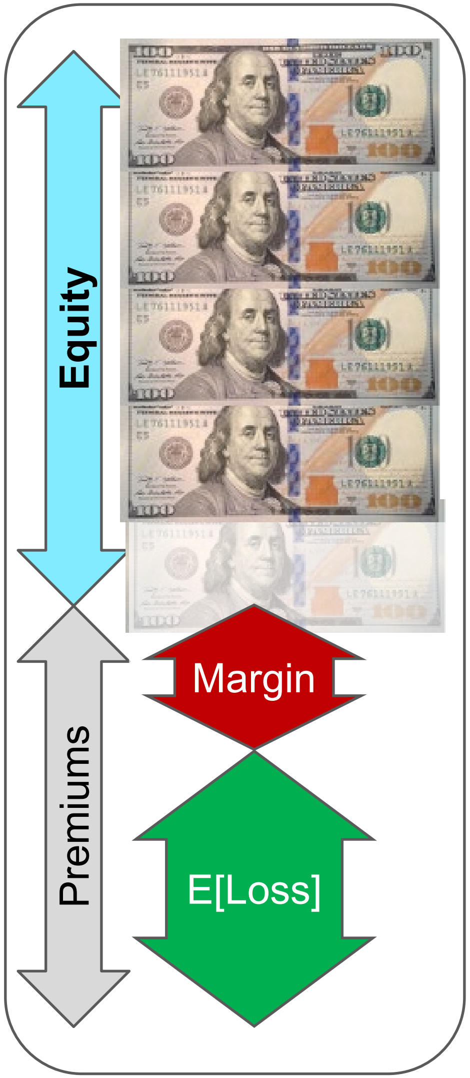
How we usually think about operations: (2) paying claims



How we usually think about operations: (3) residual value



How we usually think about operations: (4) financial reporting



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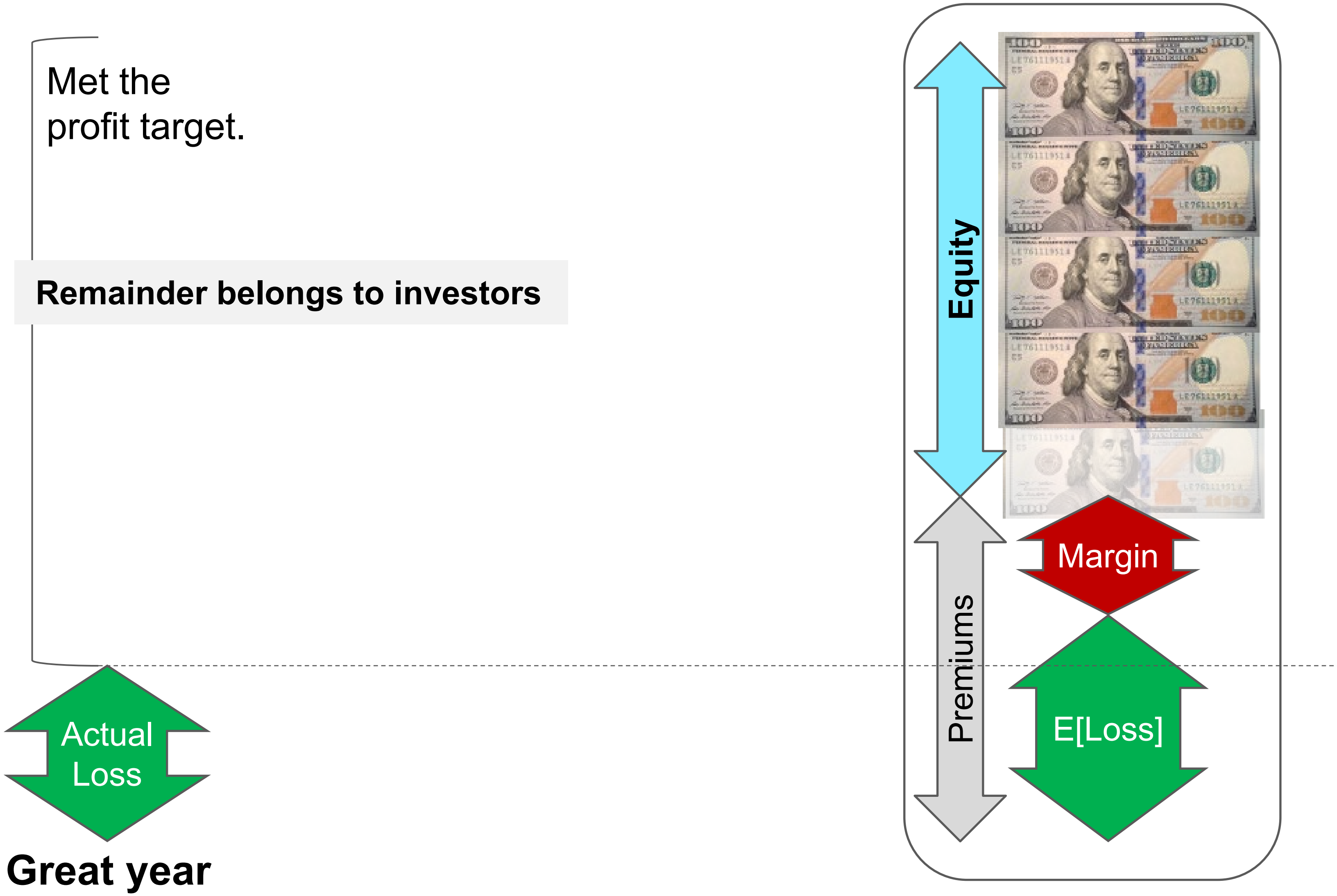

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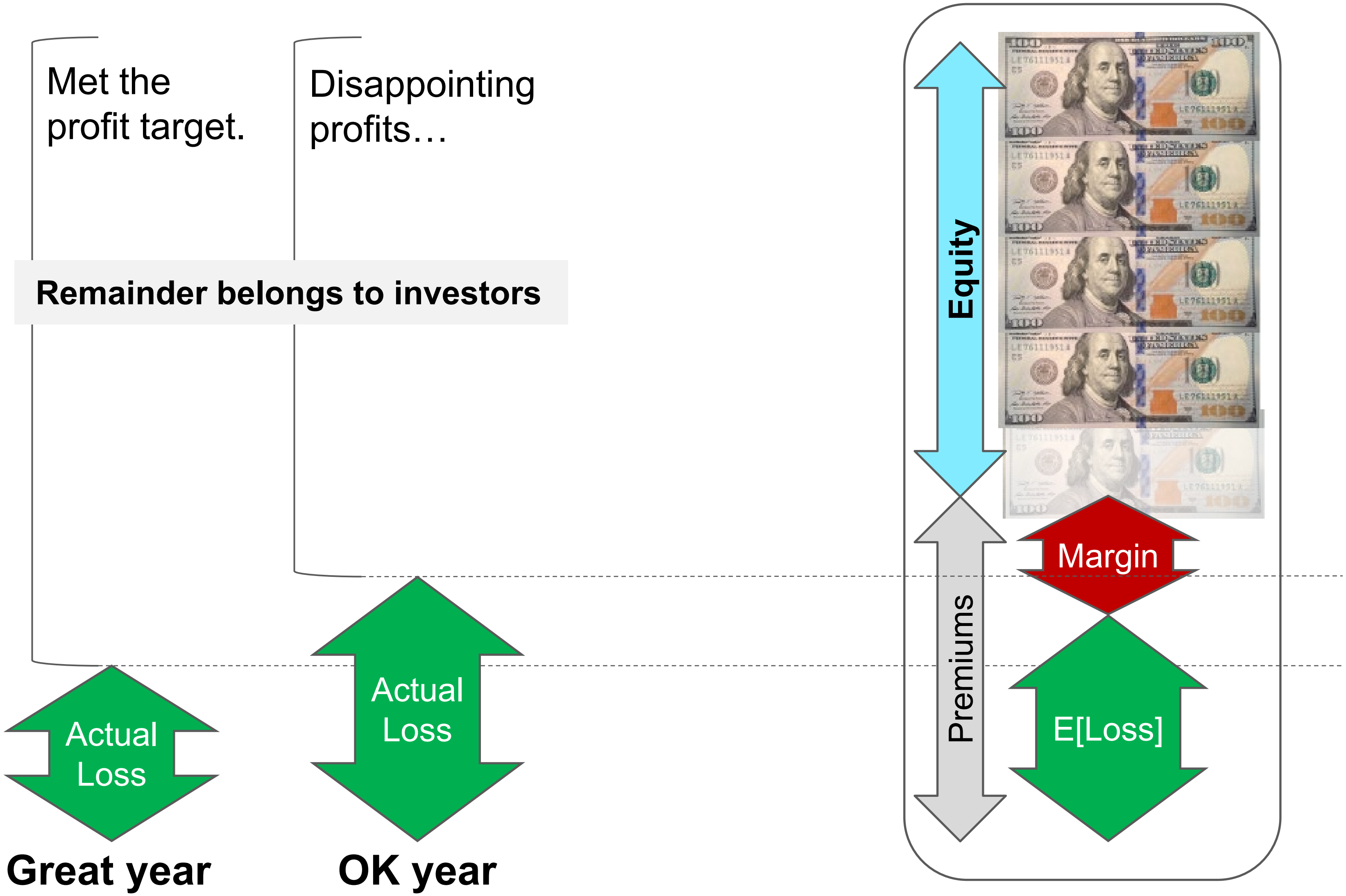

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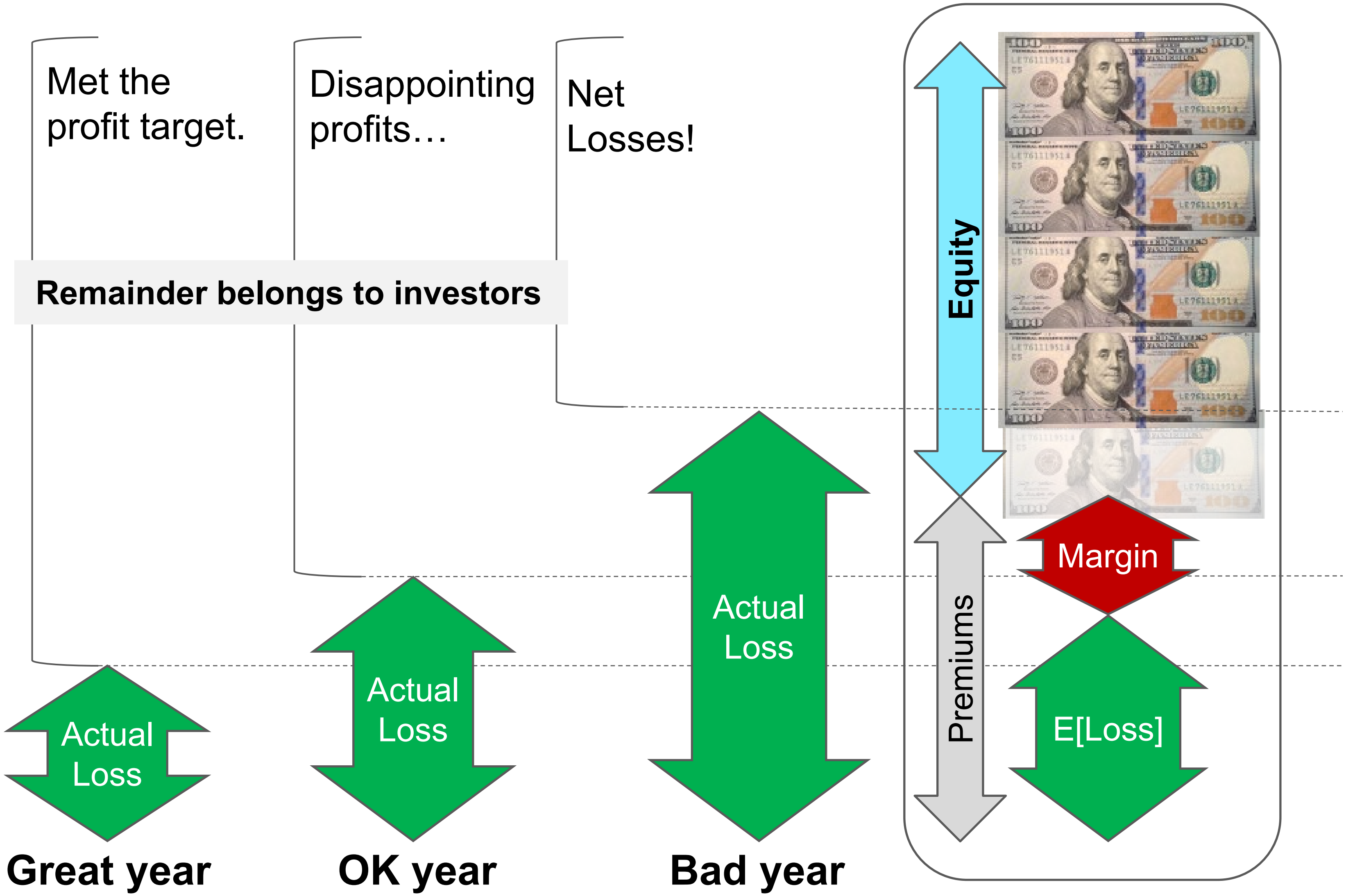

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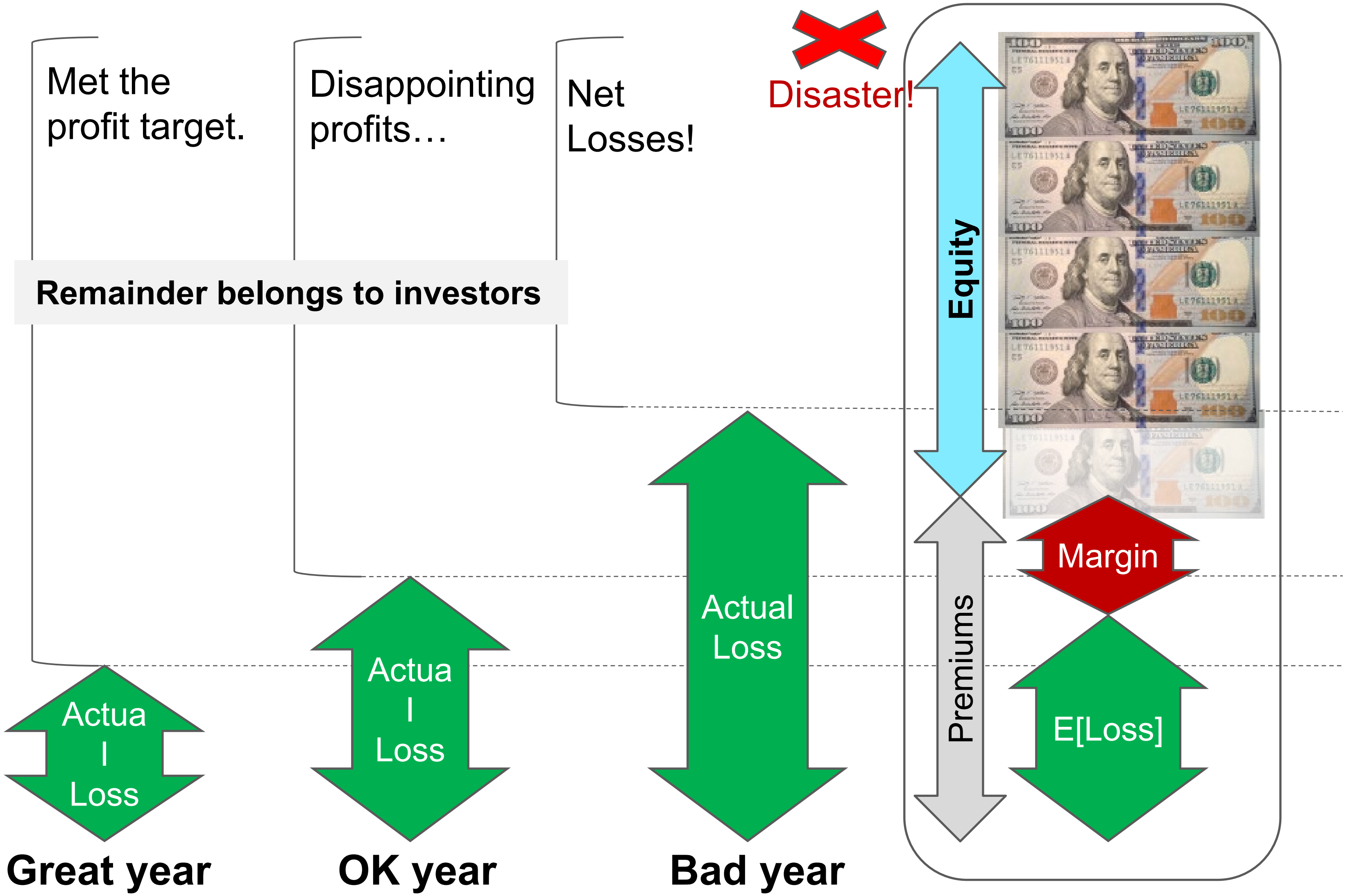
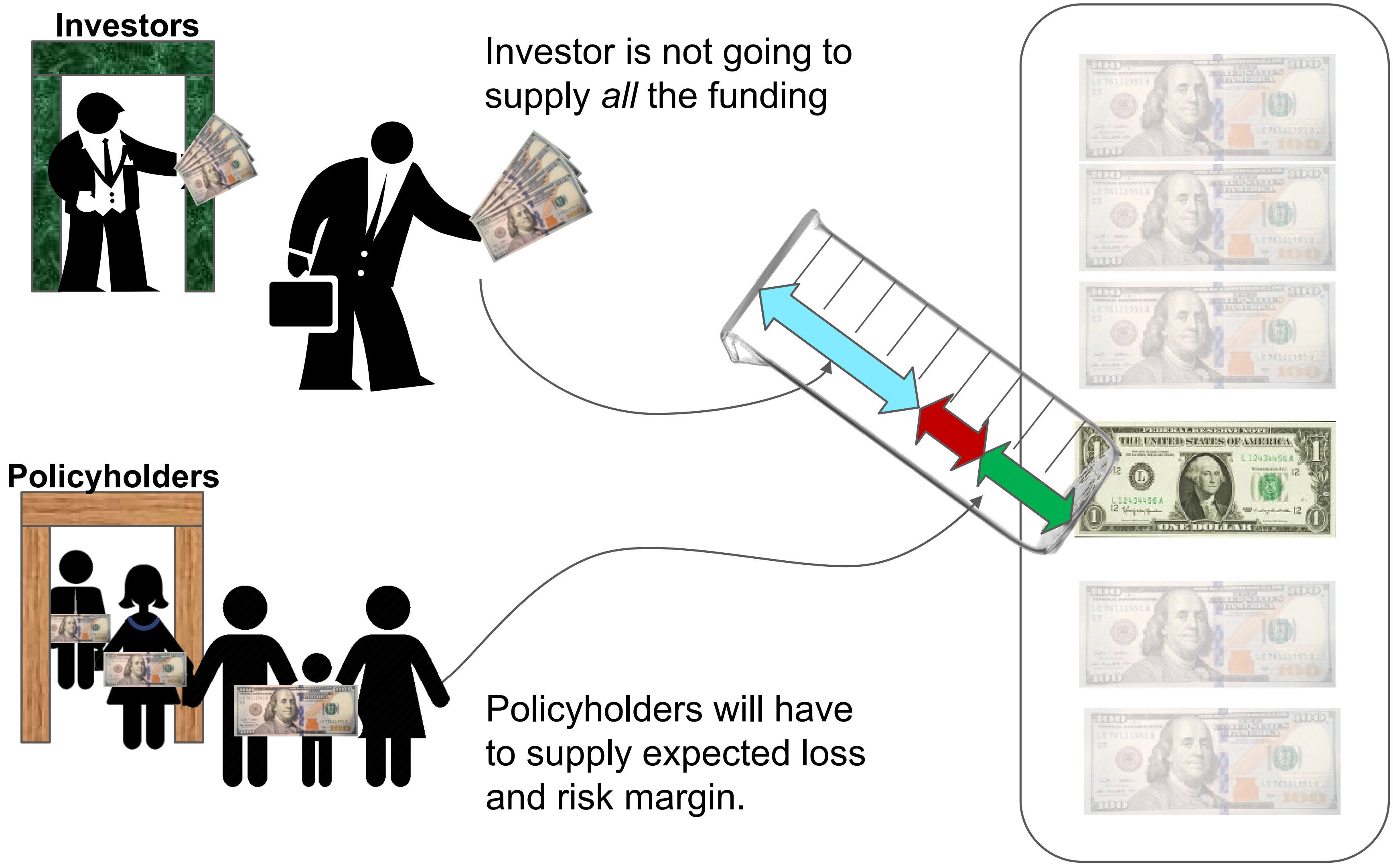


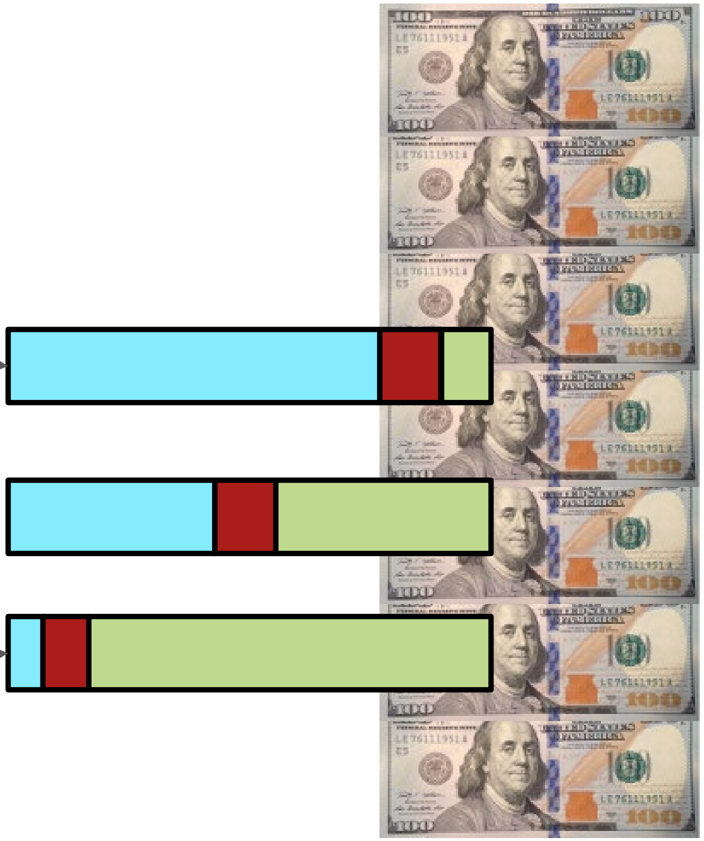
What if you had to fund each asset unit (layer) individually?



Every unit (layer) of ***asset*** has expected loss, risk margin, and equity

- Attach @ high loss
- Low probability
- High residual value

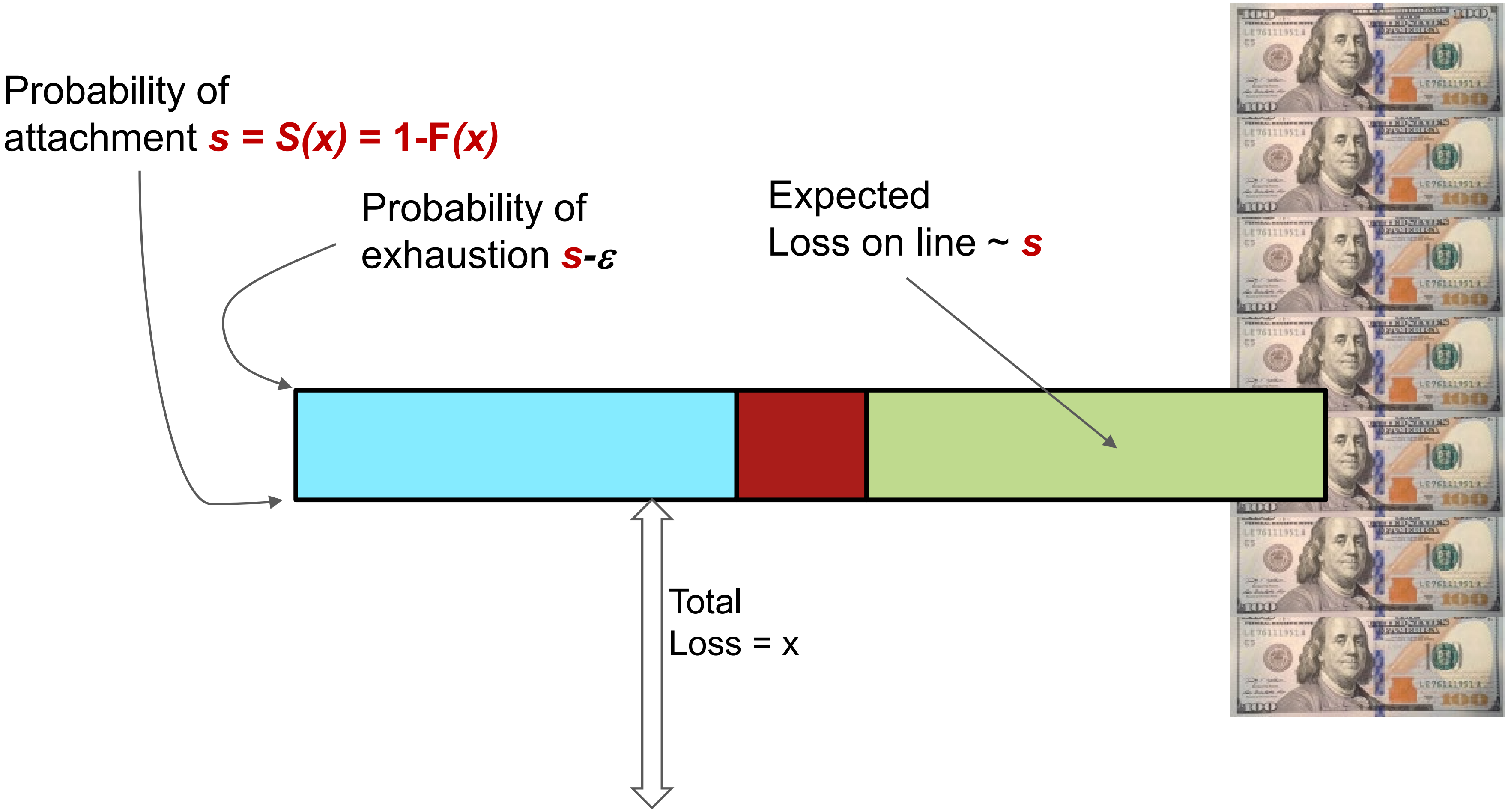
- Low premium
- High equity



- Attach @ low loss
- High probability
- Low residual value

- High premium
- Low equity

What do we know about a thin layer on the portfolio aggregate loss?

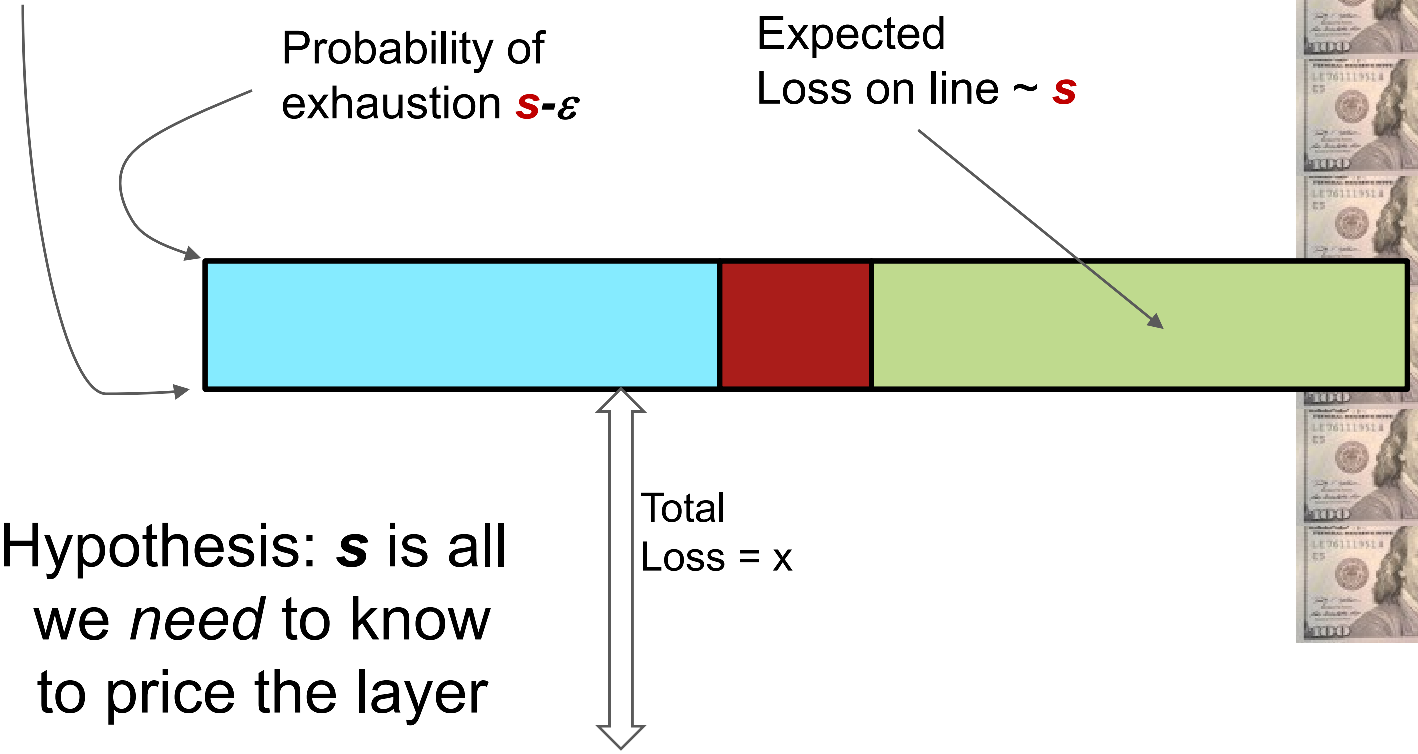


What do we know about a thin layer on the portfolio aggregate loss?

Probability of attachment ***s* = *S*(*x*) = 1-F(*x*)**

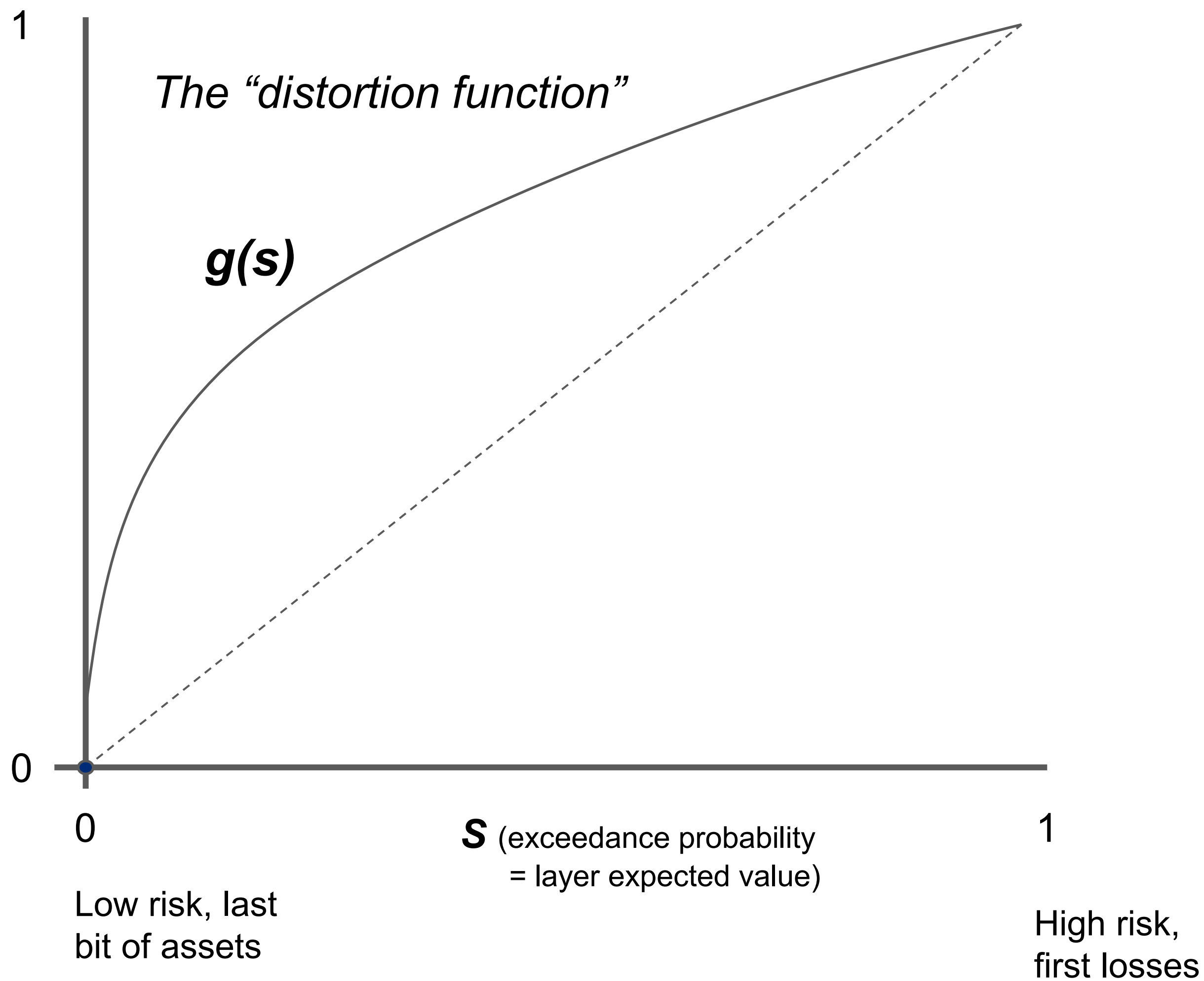
Probability of exhaustion ***s*-*ε***

Expected Loss on line ~ ***s***



Hypothesis: ***s*** is all we *need* to know to price the layer

Assumption: there is a functional relationship between layer LOL and layer ROL



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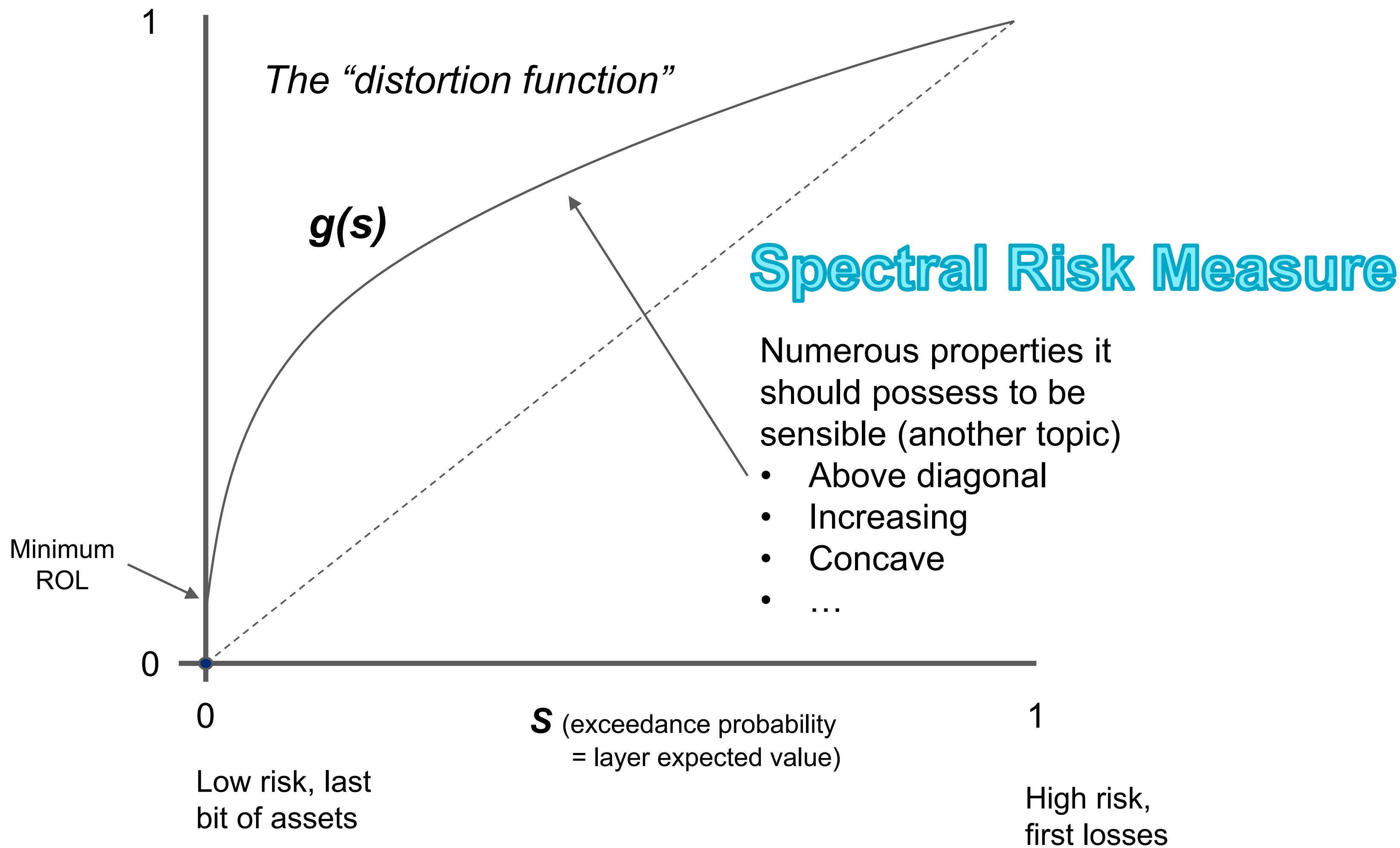

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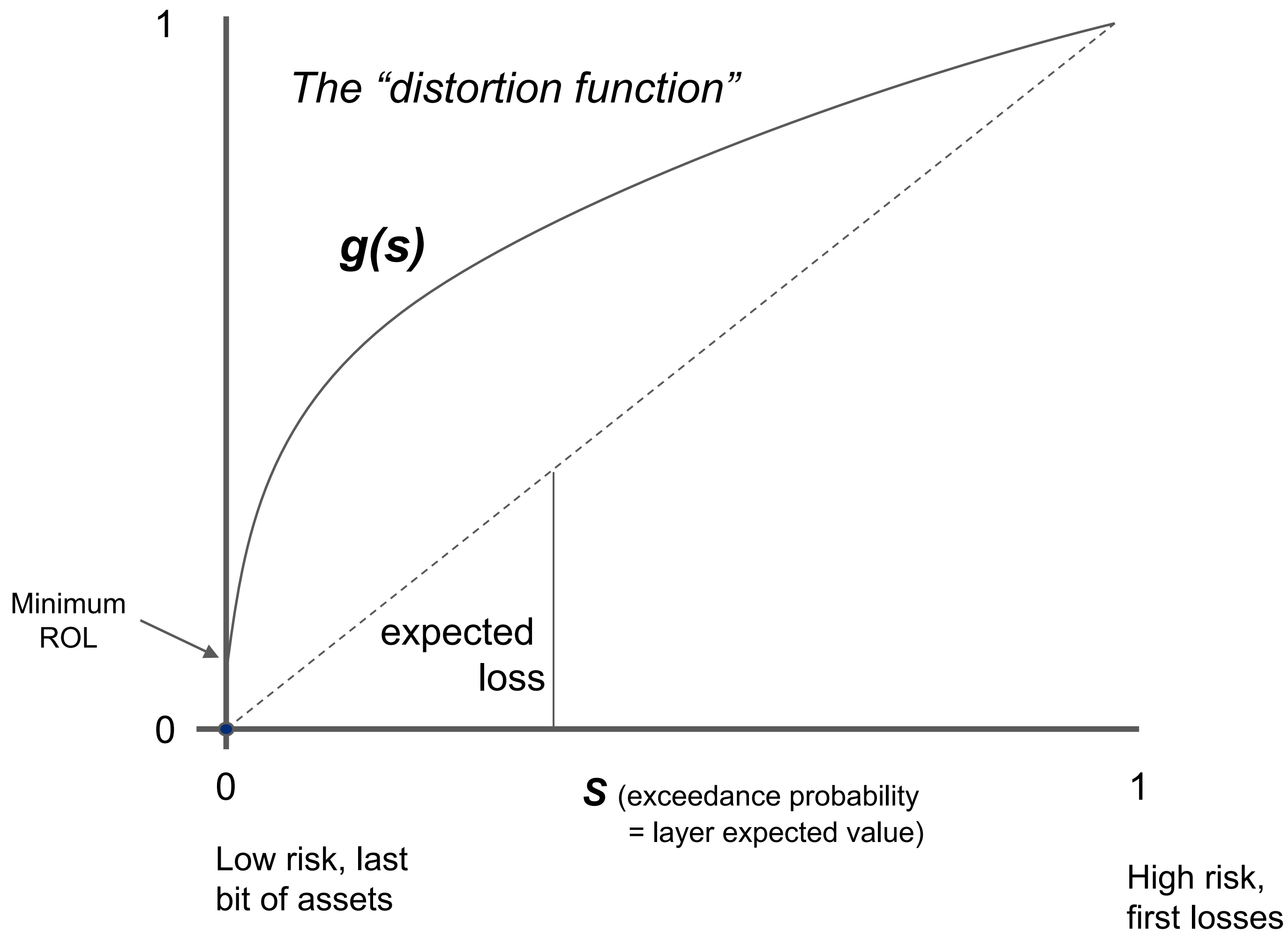

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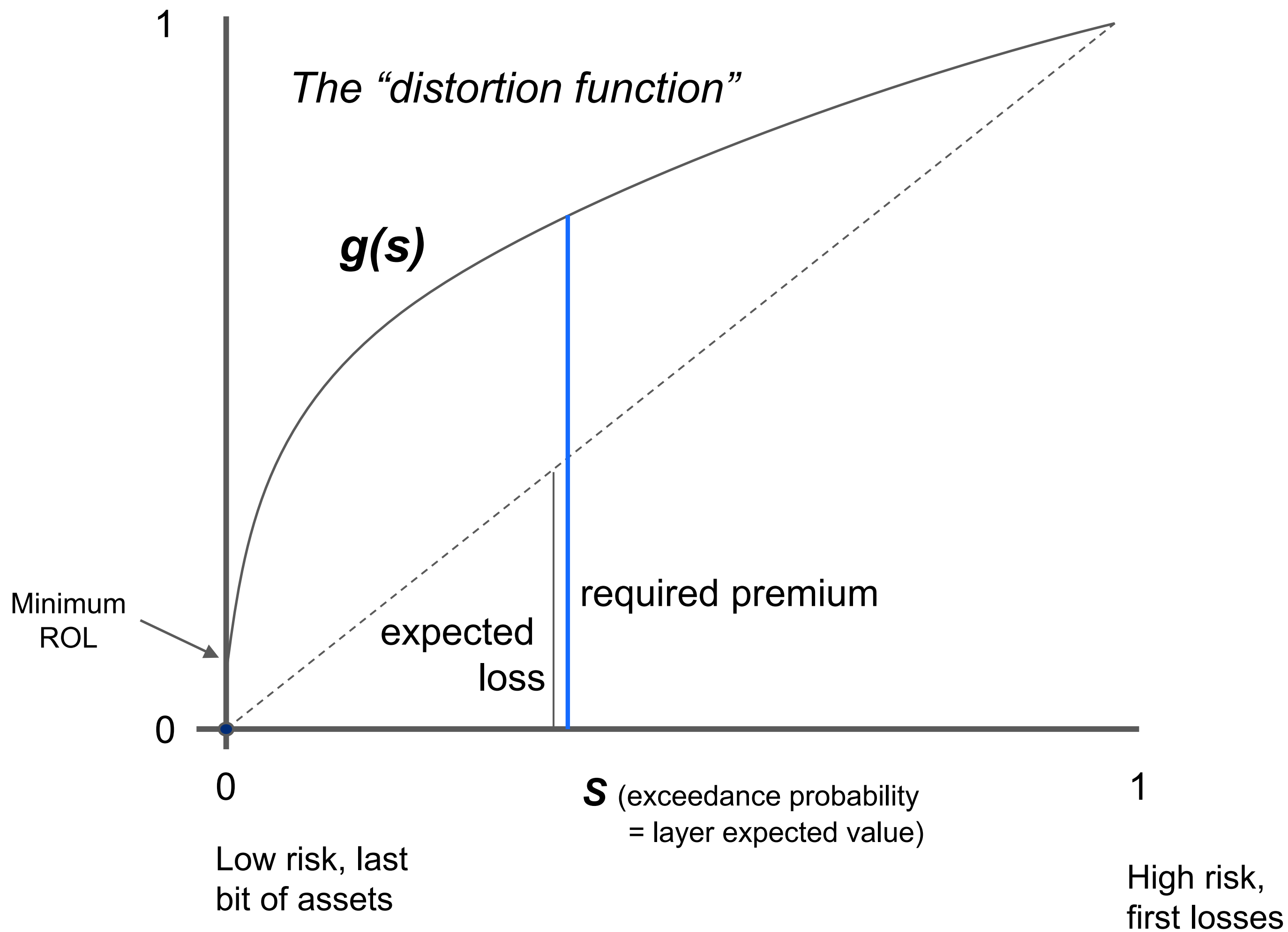
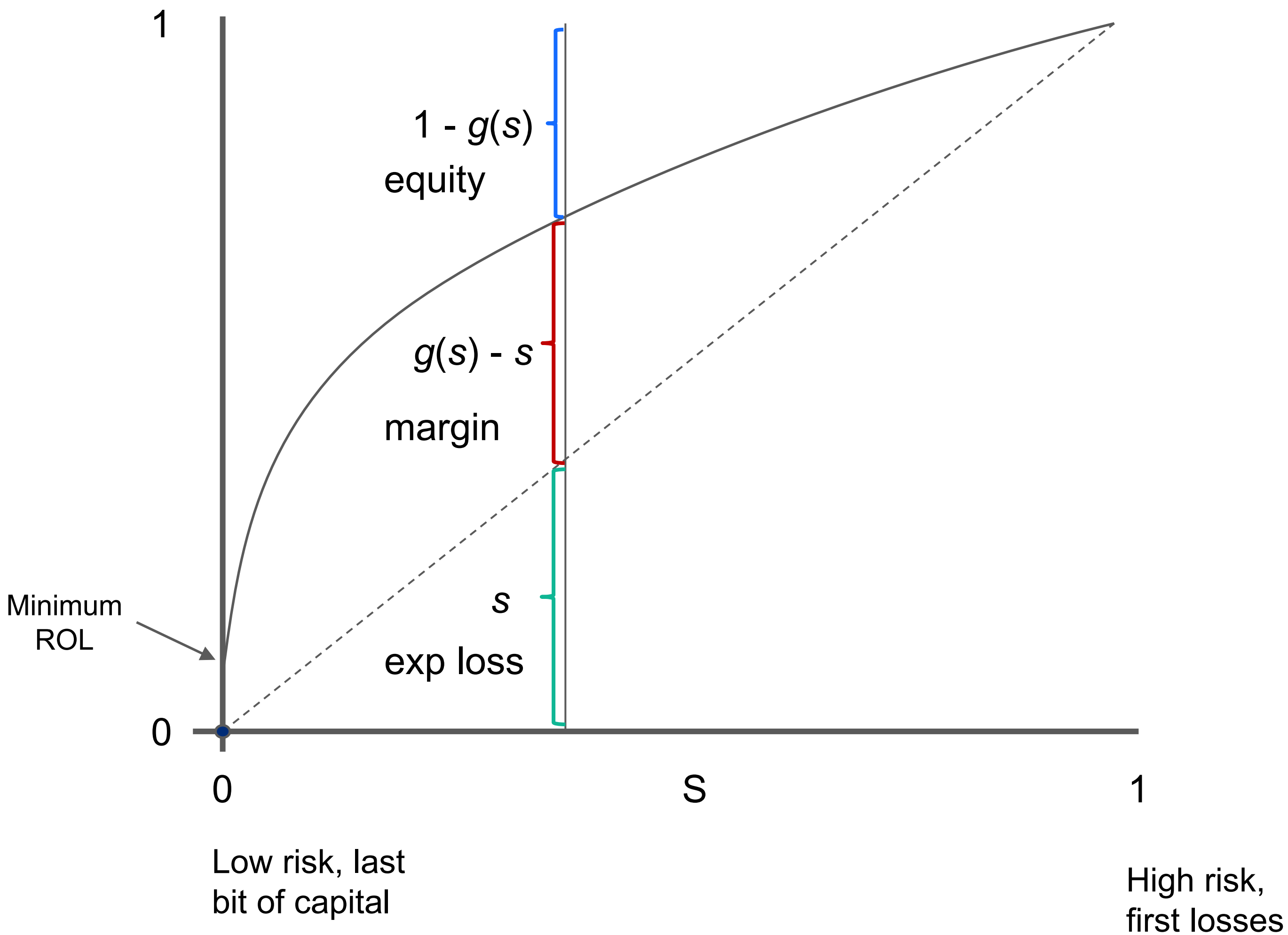


Distortion function gives you everything you want to know



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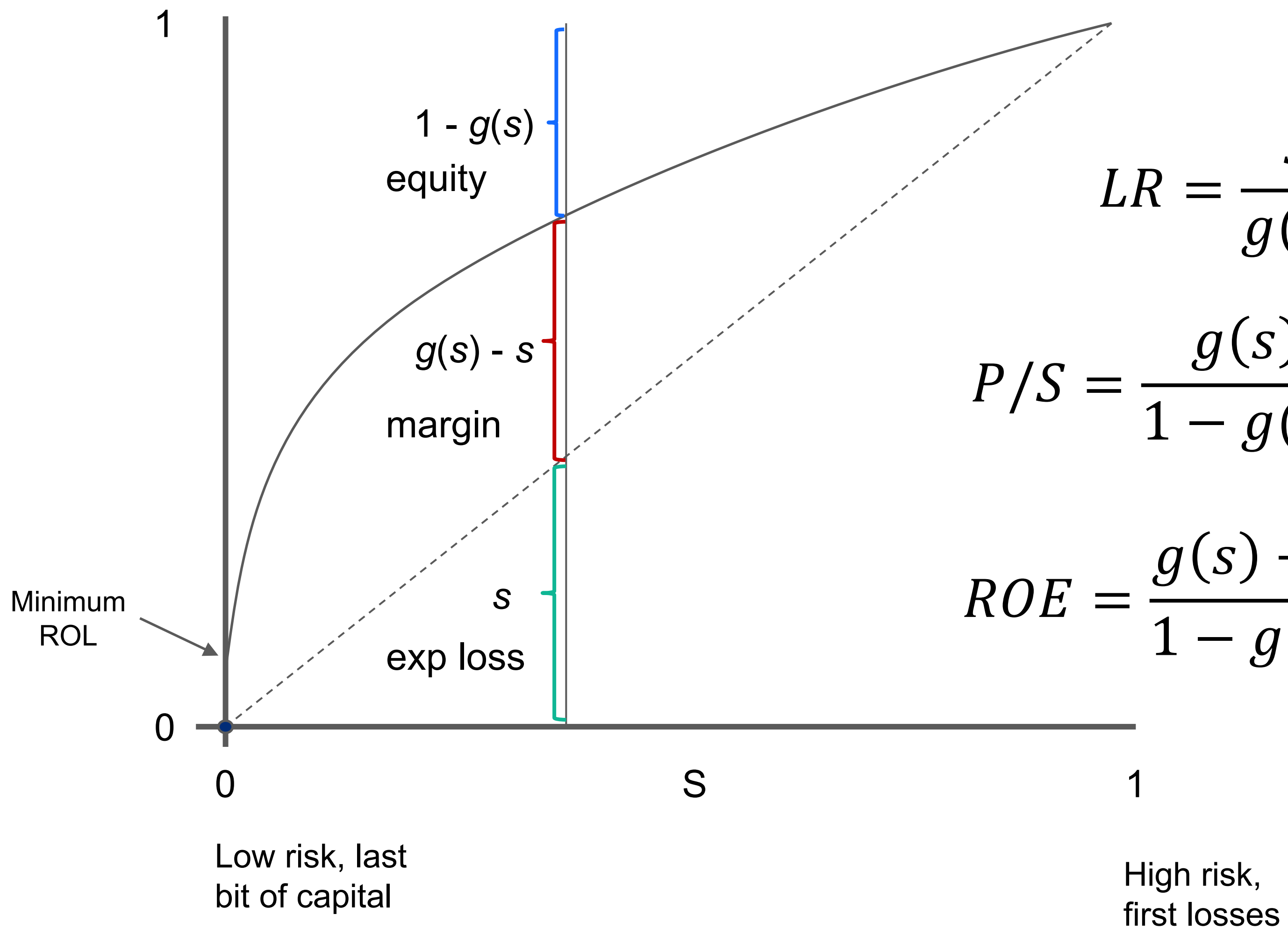
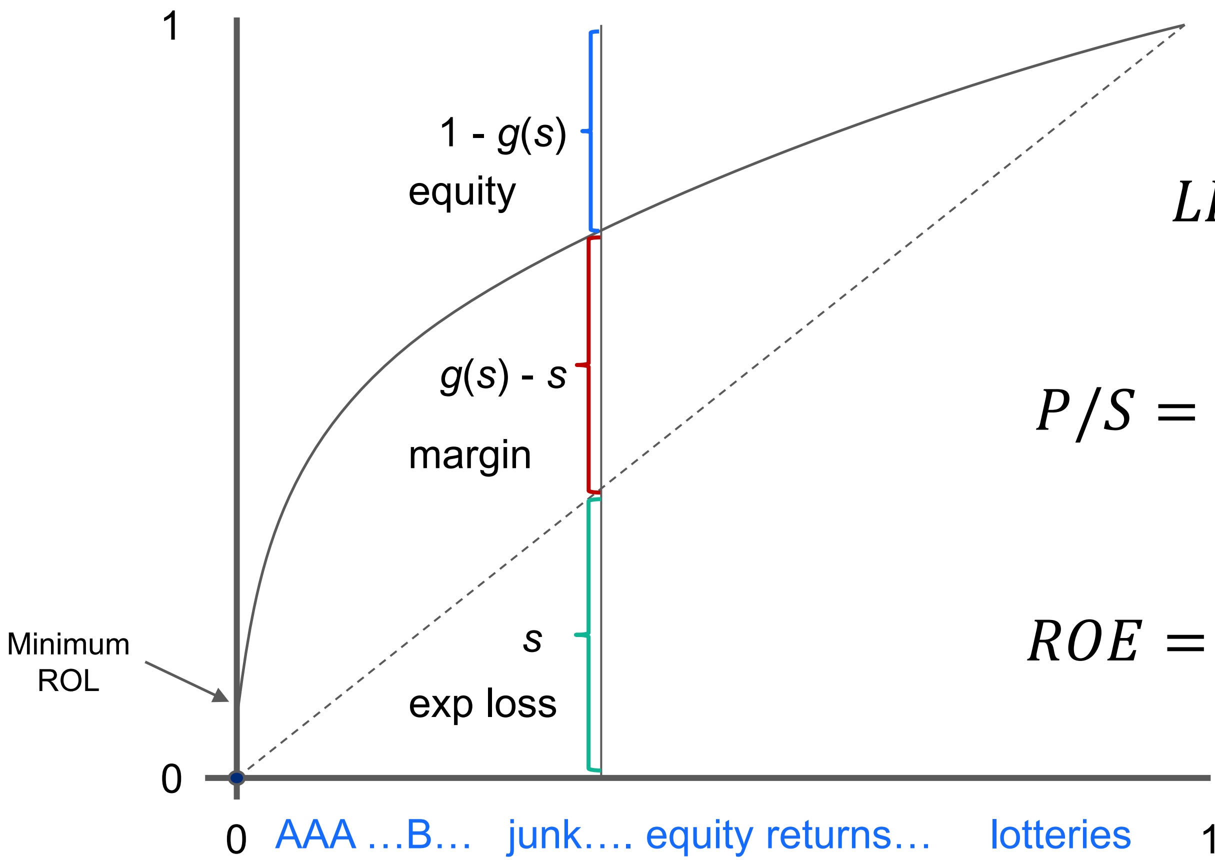


$$LR = \frac{s}{g(s)}$$

$$P/S = \frac{g(s)}{1 - g(s)}$$

$$ROE = \frac{g(s) - s}{1 - g(s)}$$

Distortion function gives you everything you want to know



Low risk, last
bit of capital

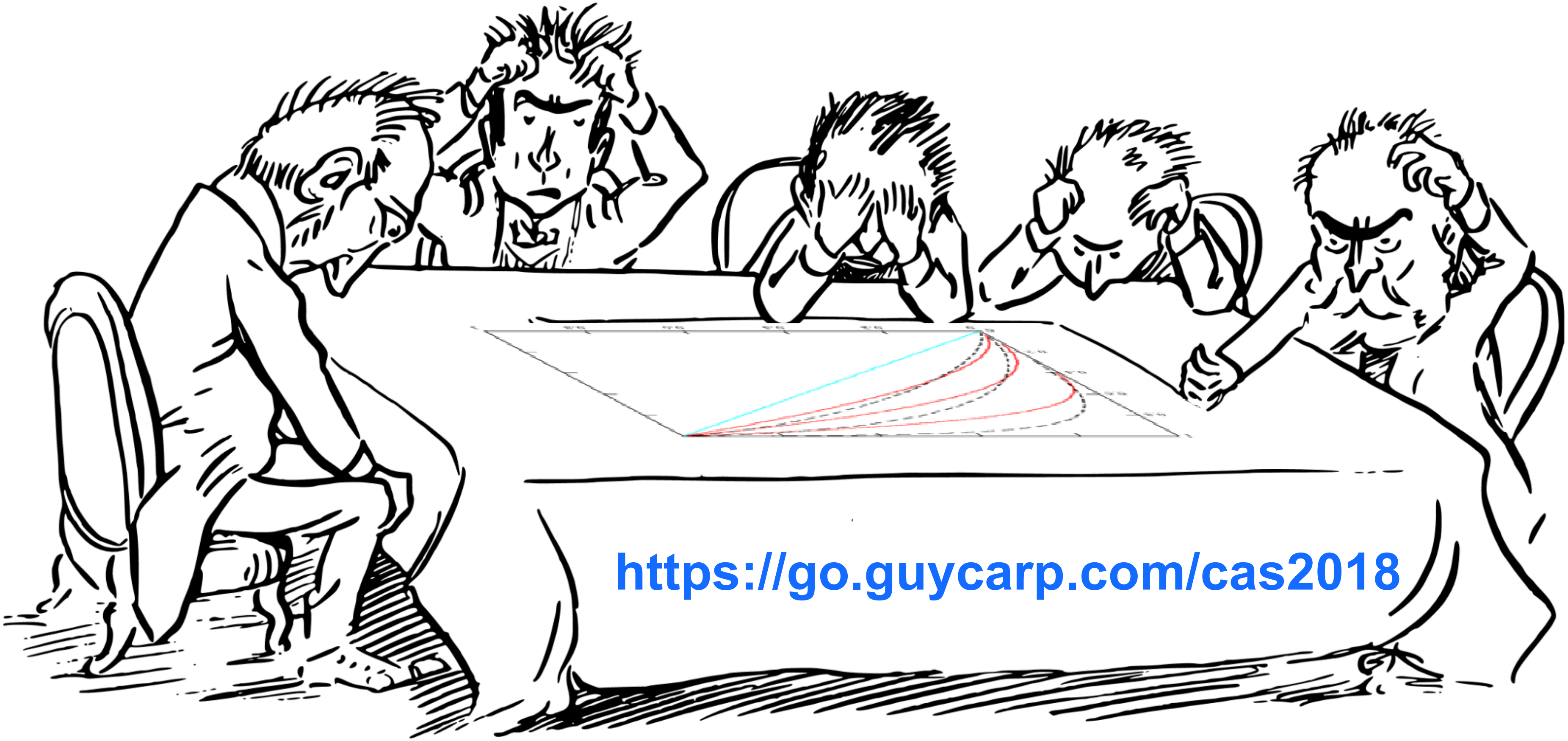
High risk,
first losses

$$LR = \frac{s}{g(s)}$$

$$P/S = \frac{g(s)}{1 - g(s)}$$

$$ROE = \frac{g(s) - s}{1 - g(s)}$$

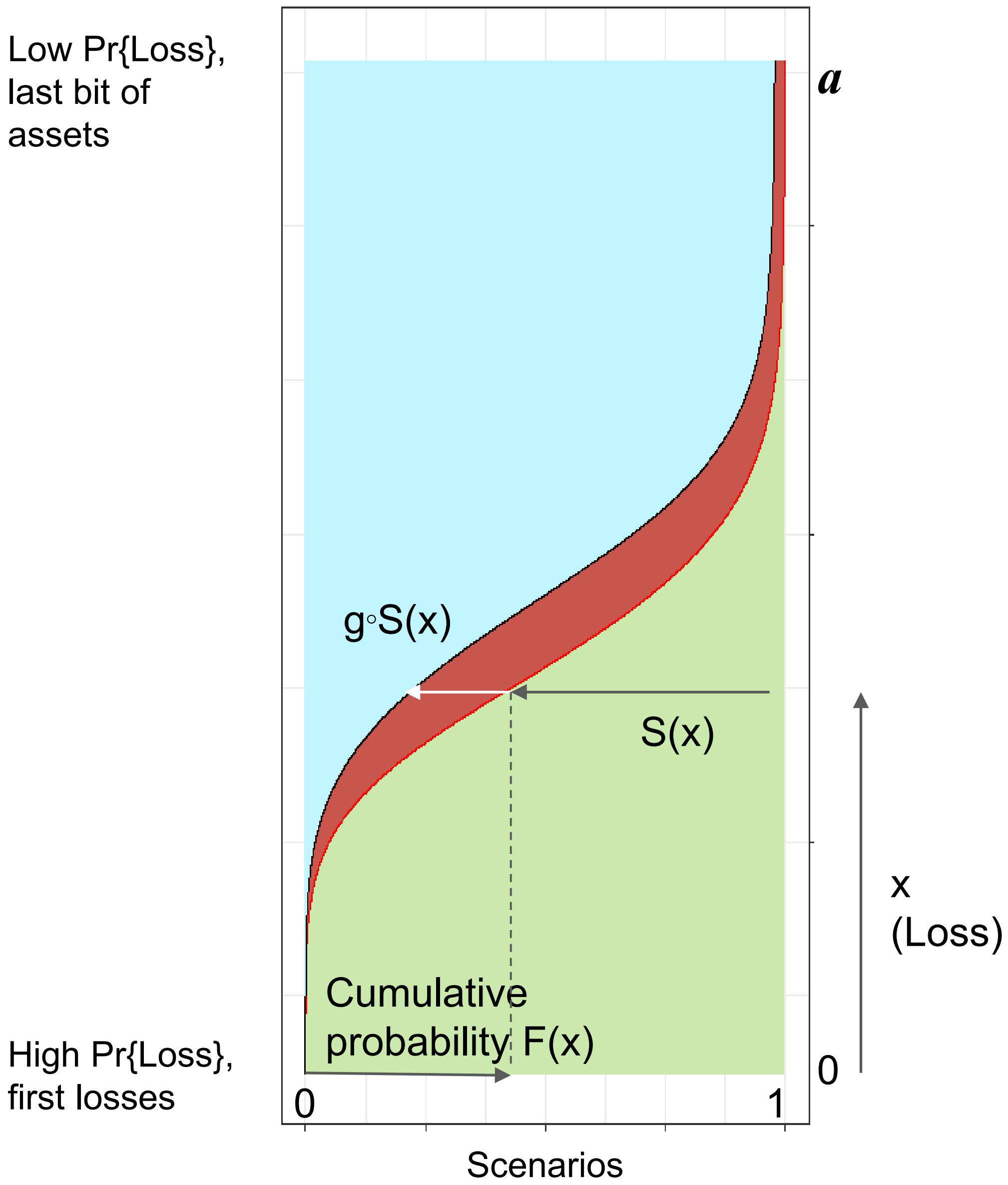
Where does that *g*(*s*) function come from?



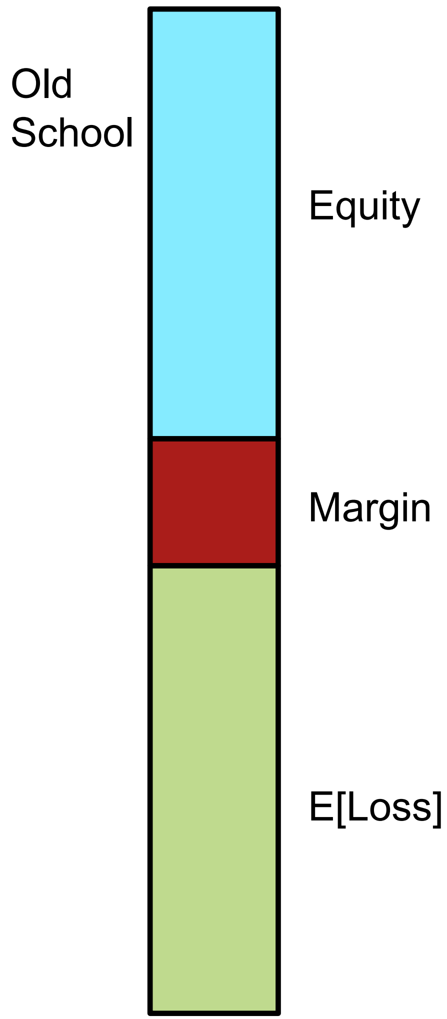
Short answer: “Financial Considerations”

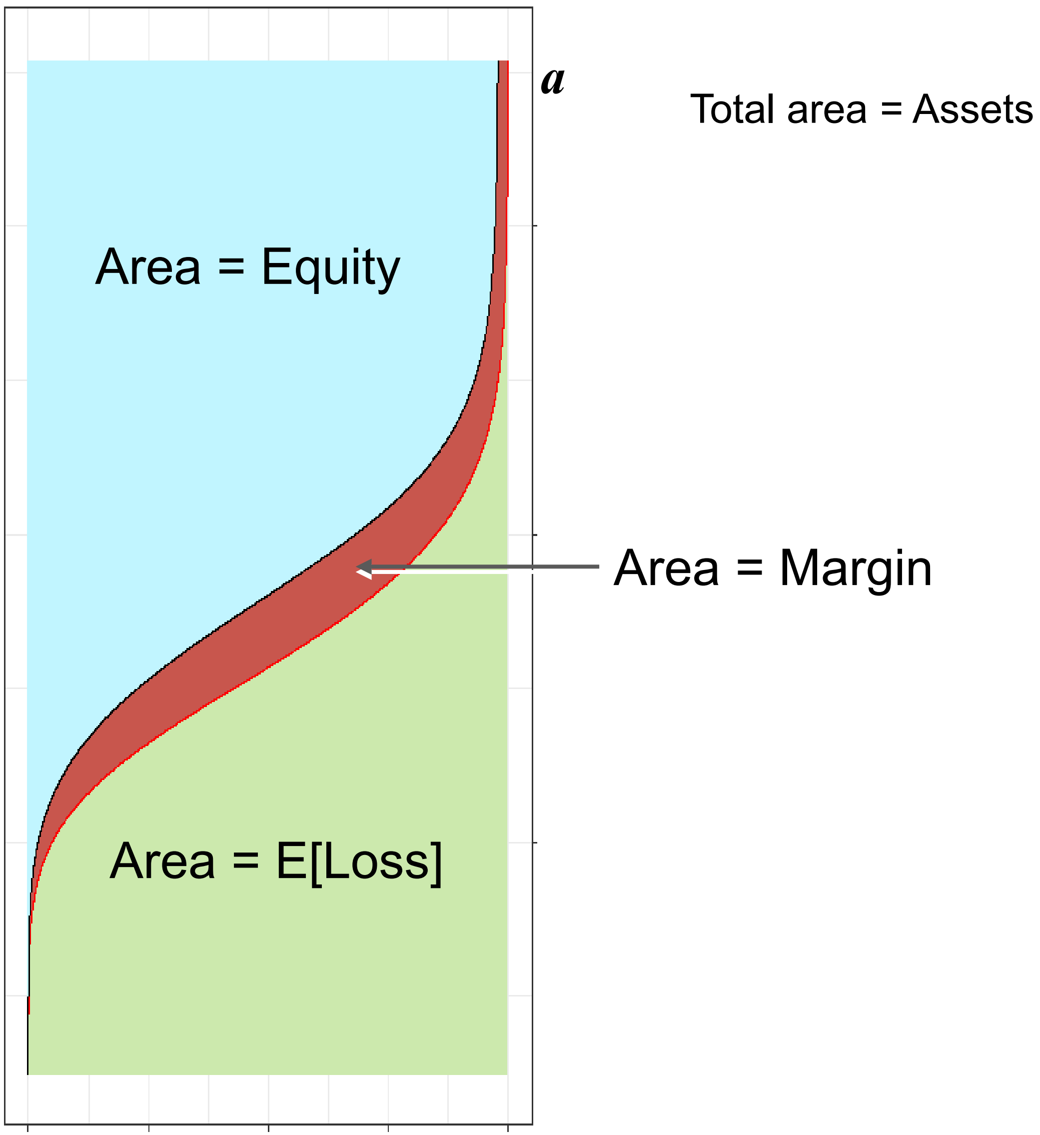
(another topic)

How it looks back in the scenario-loss domain

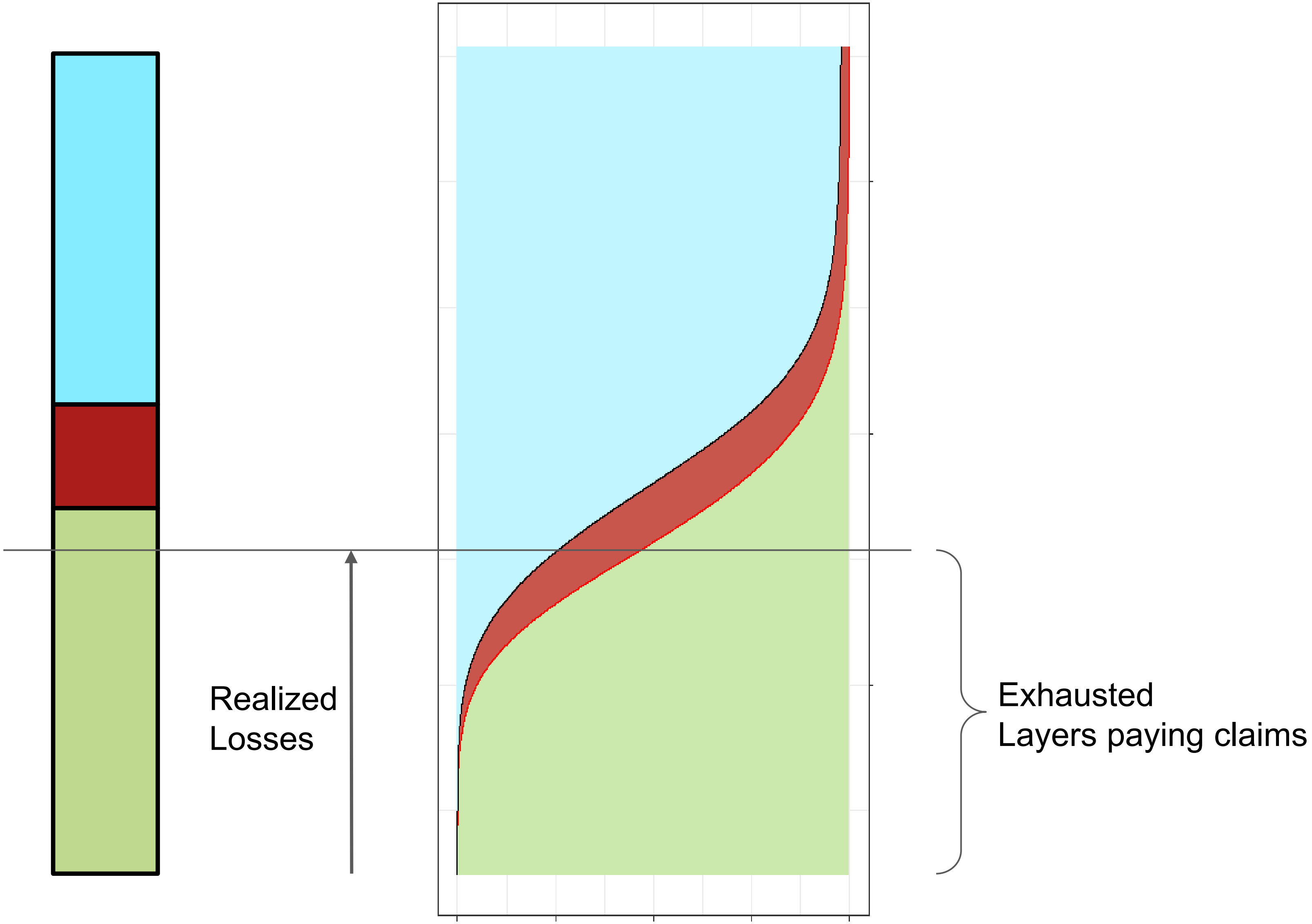


The new perspective on where premium and equity sit





After operations financial reporting



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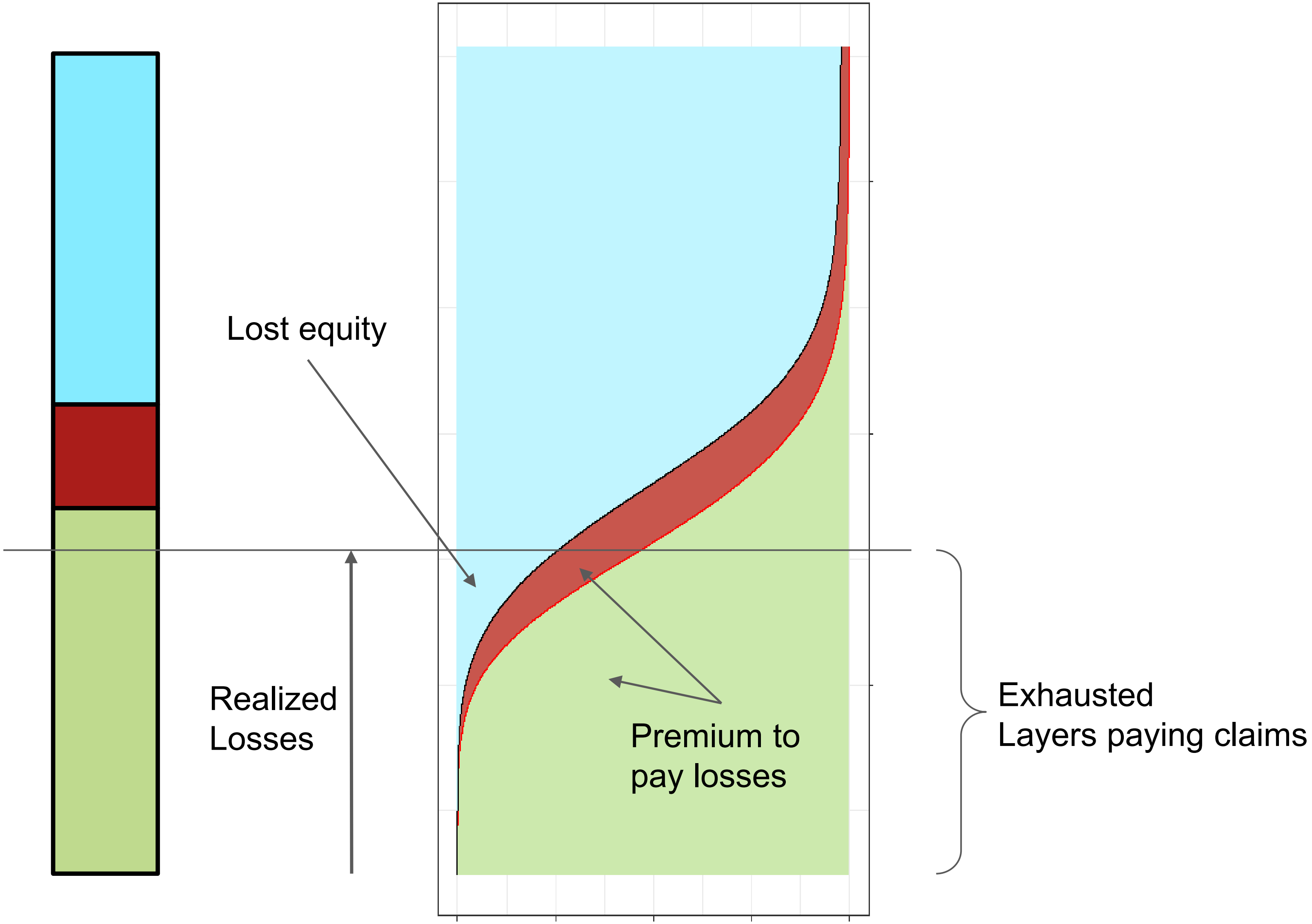

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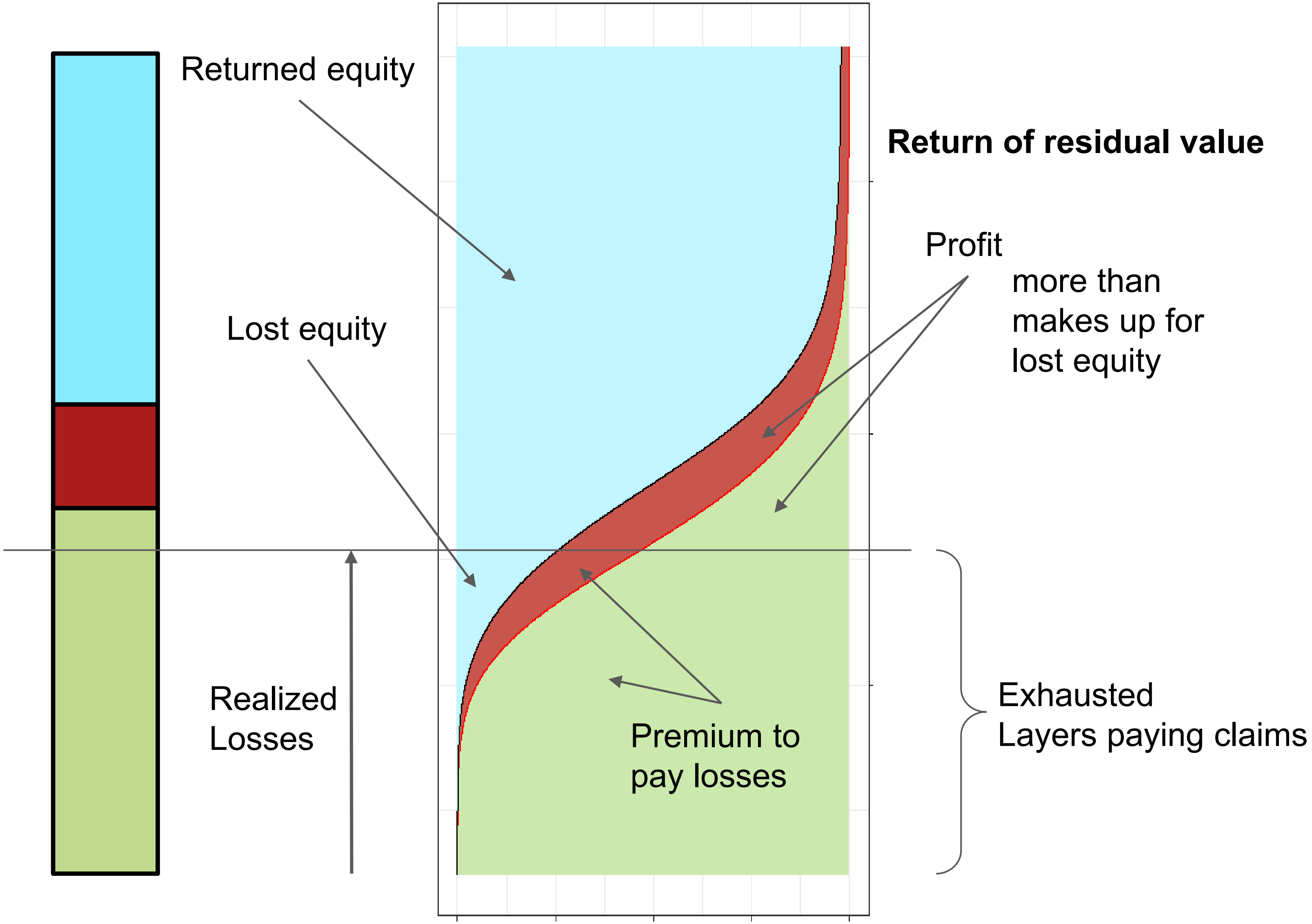
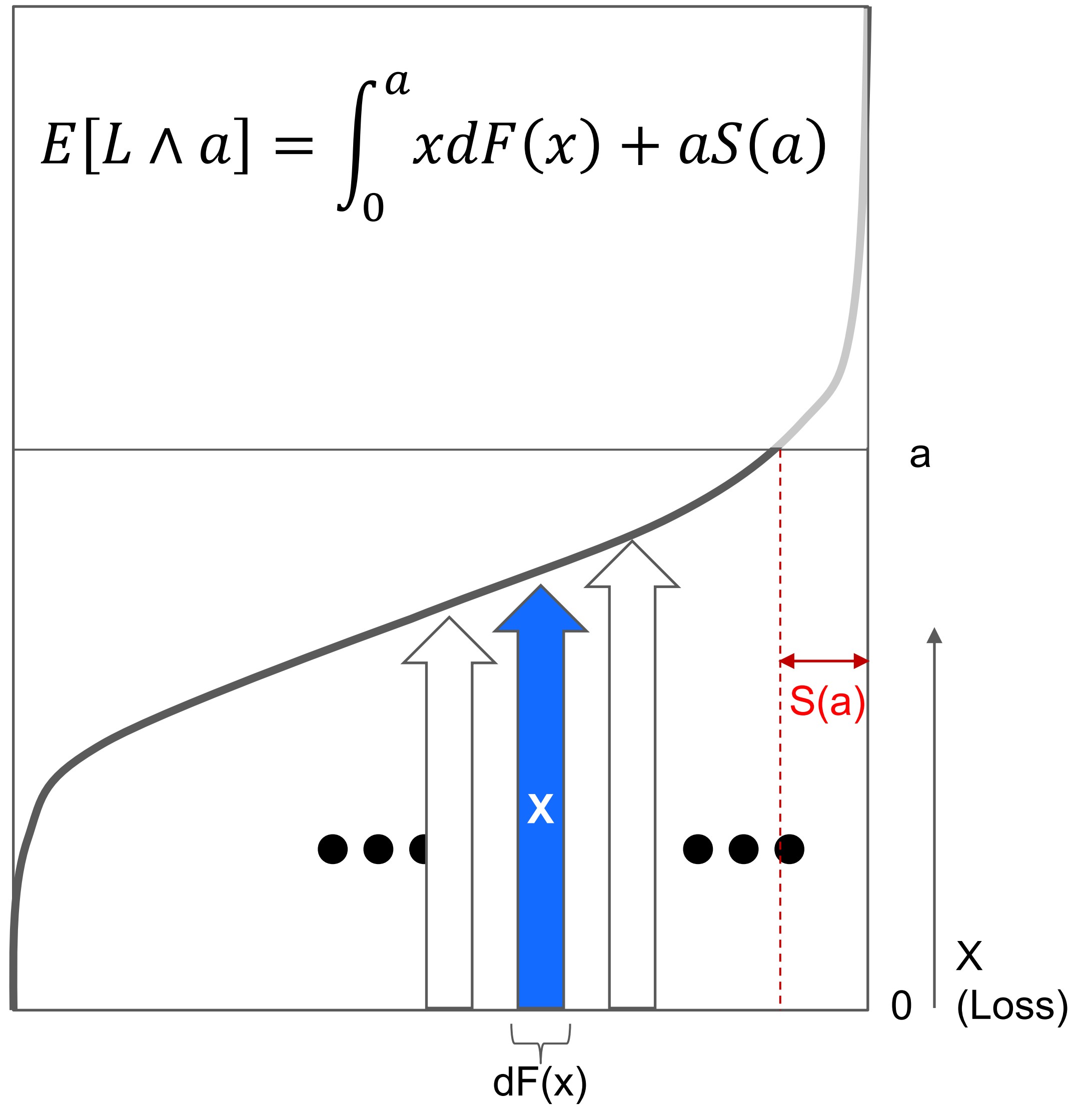
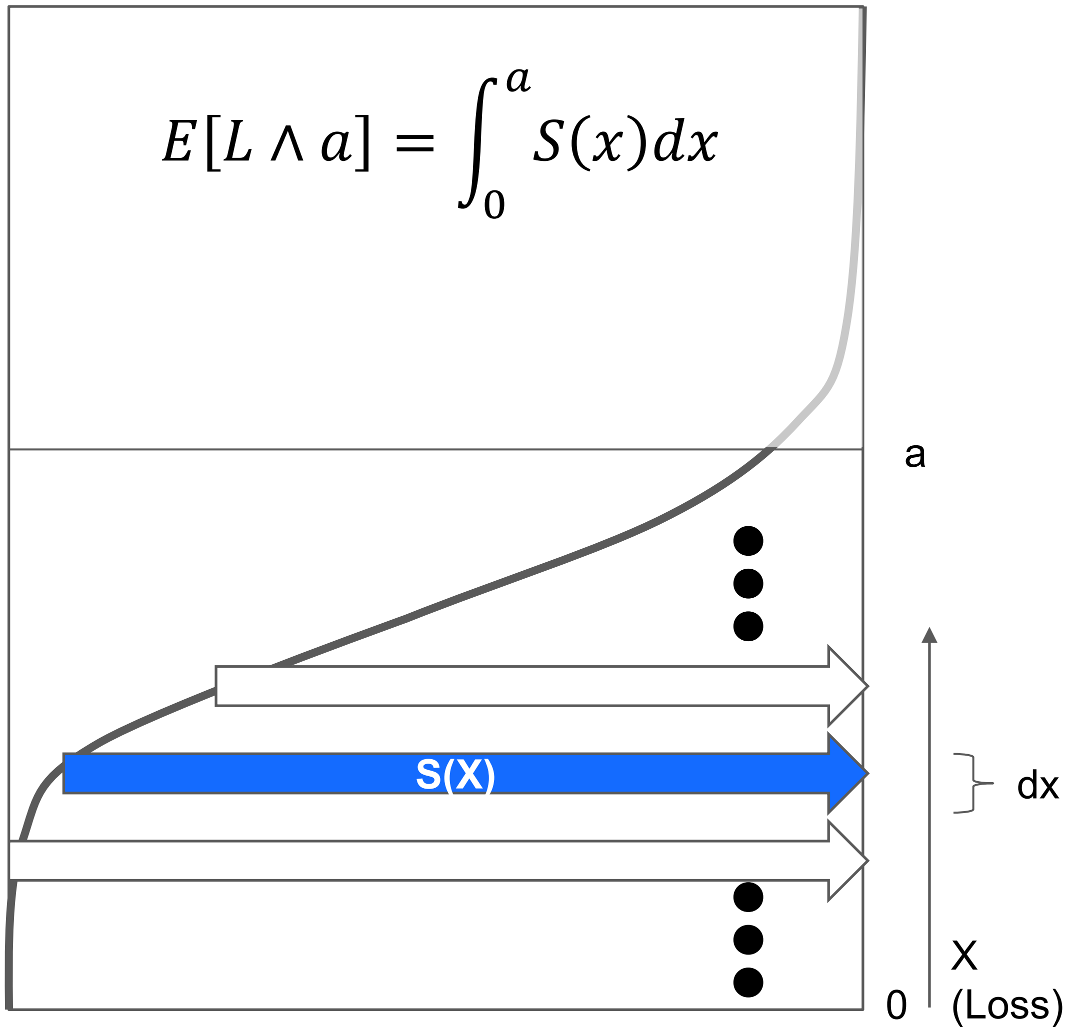


Visualizing the expectation – conventional view



Visualizing the expectation – layer view



Probability distortion implies pricing

Expected loss (LEV)

$$E[L \wedge a] = \int_0^a S(x)dx = \int_0^a x dF(x) + aS(a)$$

Probability distortion implies pricing

Expected loss
(LEV)

$$E[L \wedge a] = \int_0^a S(x)dx = \int_0^a x dF(x) + aS(a)$$

distorted probability   transformed cdf

Required premium
Distorted expected loss

$$E_g[L \wedge a] = \int_0^a g\big(S(x)\big)dx = \int_0^a x dG(x) + ag\big(S(a)\big)$$

Pause

- What you’ve seen so far:
  - Thinking about layers of assets
  - Each consists of premium + equity
  - Margin is cost of capital
  - Expected loss ***s*** determines layer funding
  - Functional relationship ***g*(*s*)**
  - … leads to Spectral Risk Measure => pricing

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convex risk

Spectral Risk Measures and Pricing Insurance Risk: Part 2

Stephen J. Mildenhall

CAS Spring Meeting, May 13, 2020

Loss payments: who gets what in default?

- Sold insurance promises

$$X = X_1 + \cdots + X_{\mathrm{n}}$$

- **Equal priority** payment to line *i* with assets *a*

$$X_i(a) = \begin{cases} X_i & X \le a \\ a\ (X_i/X) & X > a \end{cases}$$
$$= X_i \frac{X \wedge a}{X}$$
$$= \frac{X_i}{X} X \wedge a$$

- $\dfrac{X \wedge a}{X}$ = fixed payment pro rata factor applied to loss from all lines
- $\dfrac{X_i}{X}$ = variable share of available assets for line *i*
- $X \wedge a$ amount of assets **available** to pay claims
- $X_i(a)$ sum to $X \wedge a$, limited losses

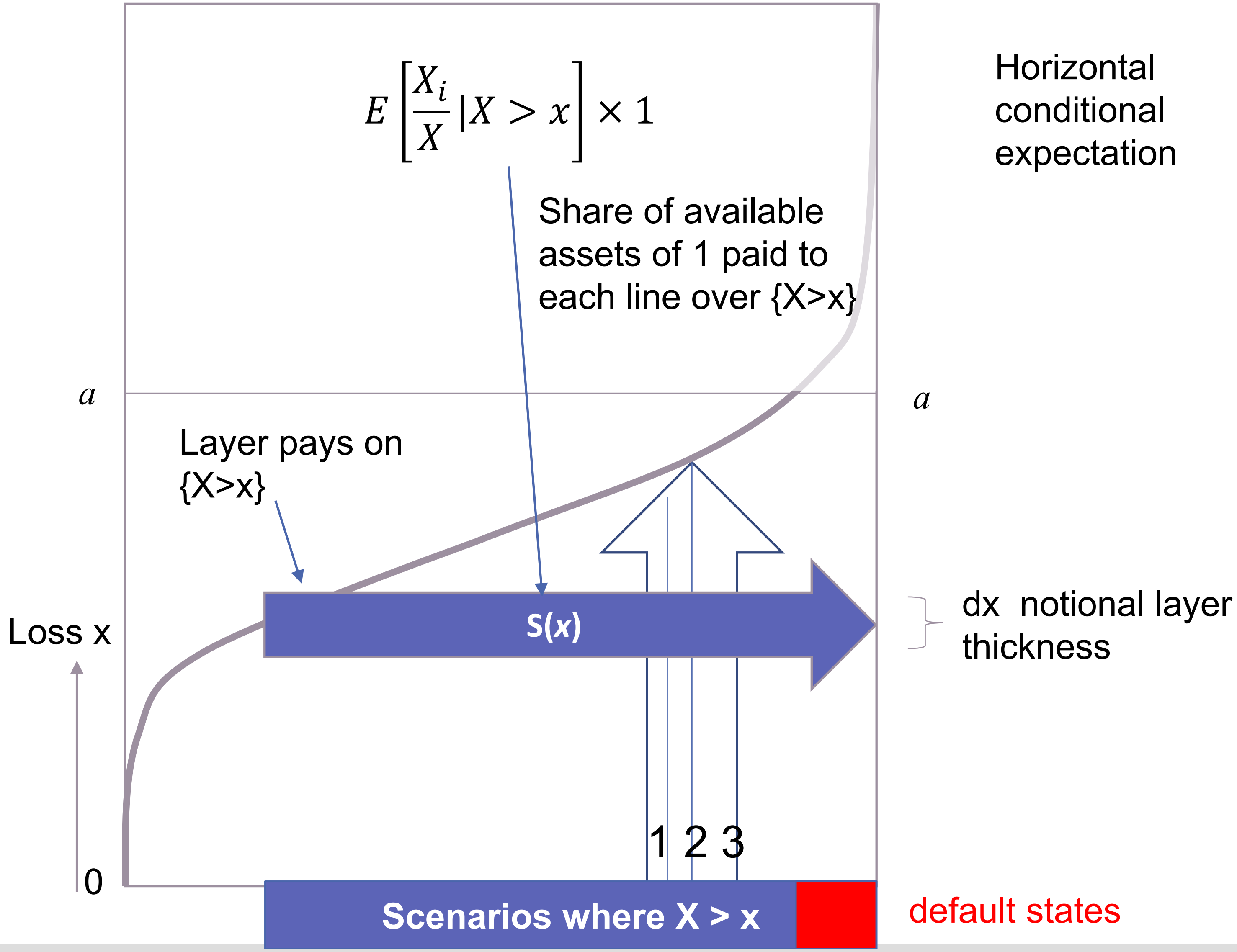
convex risk

Expected loss formulas

$$E[X \wedge a] = \int_0^a S(x)dx$$

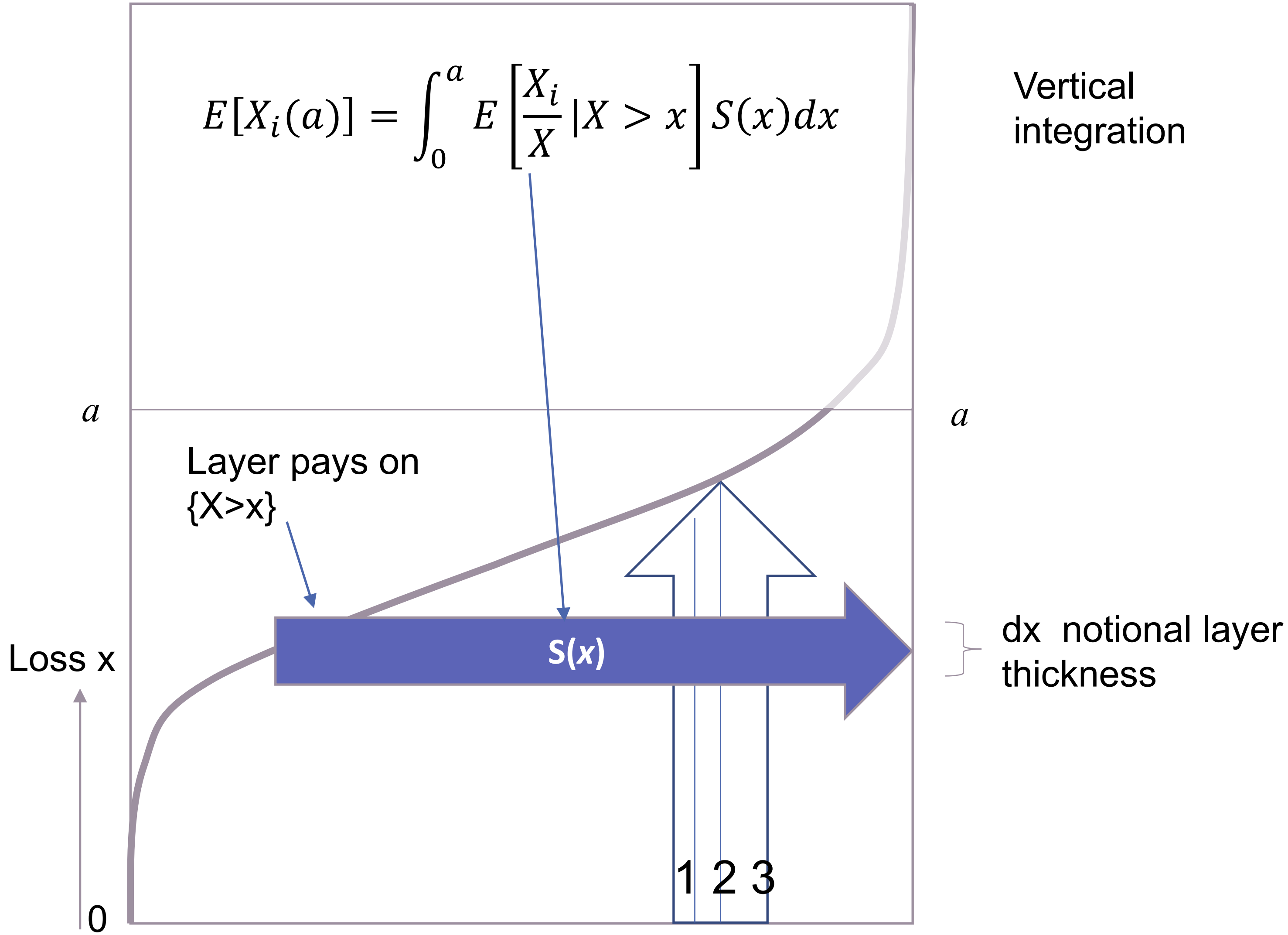
$$E[X_i(a)] = ??$$

Visualizing expected loss by line and layer and total



Visualizing expected loss by line and layer and total

$$E[X_i(a)] = \int_0^a E\left[\frac{X_i}{X} | X > x\right] S(x)dx$$



Expected loss and premium by line and layer and total

$$\bar{L}_i(a) = E[X_i(a)] = \int_0^a E\left[\frac{X_i}{X} | X > x\right] S(x)dx = \int_0^a \alpha_i(x)S(x)dx$$

$$\underbrace{E\left[\frac{X_i}{X} | X > x\right]}_{\alpha_i(x)}$$

$$\bar{P}_i(a) = E_g[X_i(a)] = \int_0^a E_g\left[\frac{X_i}{X} | X > x\right] g\big(S(x)\big)dx = \int_0^a \beta_i(x)g\big(S(x)\big)dx$$

$$\underbrace{E_g\left[\frac{X_i}{X} | X > x\right]}_{\beta_i(x)}$$

α_i , β_i functions add-up: $\sum \alpha_i(x) = E\left[\frac{X_1+\cdots+X_n}{X} | X > x\right] = 1$

convex risk

Expected loss and premium by line and layer and total

Loss cost density $L_i(x) = \alpha_i(x)S(x)$

Premium density $P_i(x) = \beta_i(x)g\big(S(x)\big)$

$\Rightarrow$ Margin density $M_i(x) = P_i(x) - L_i(x)$

$$= \beta_i(x)g\big(S(x)\big) - \alpha_i(x)S(x)$$

- Integrate density to get total
- Everything you need to price!
- All quantities add-up
- Not an arbitrary allocation...no choices

Assumptions

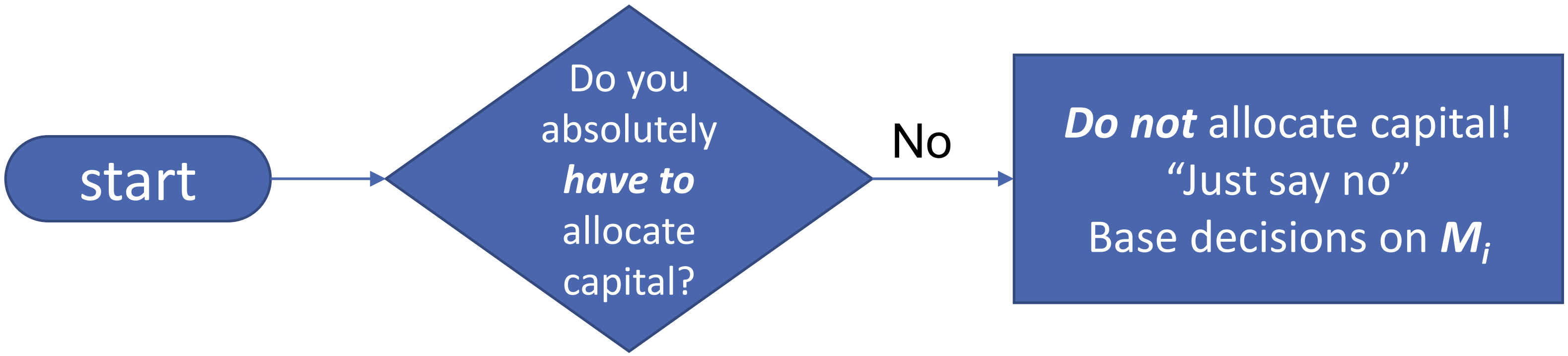
- Price with g
- Equal priority in default

Independence of X_i **not** required

Three subtle points

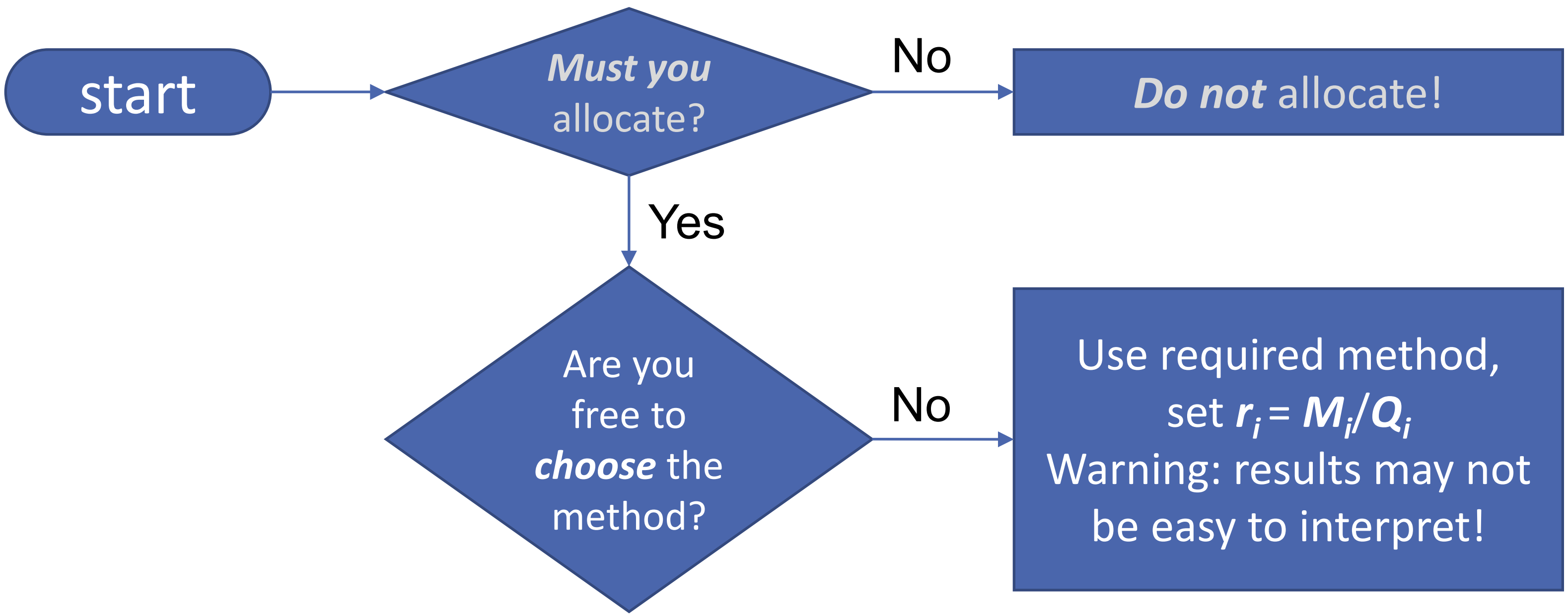
- E_g is not additive, the risk adjustment depends on X, really $\rho(X)$
- Allocation of an allocation: is risk adjustment based on X or $X \wedge a$?
  - It can matter...it doesn't for SRMs
  - Comonotonic additive
- Non-uniqueness: is risk adjustment (conditional measure) unique?
  - No...but it doesn't matter for SRMs
  - Law invariant and comonotonic additive

Capital allocation recommendation flowchart



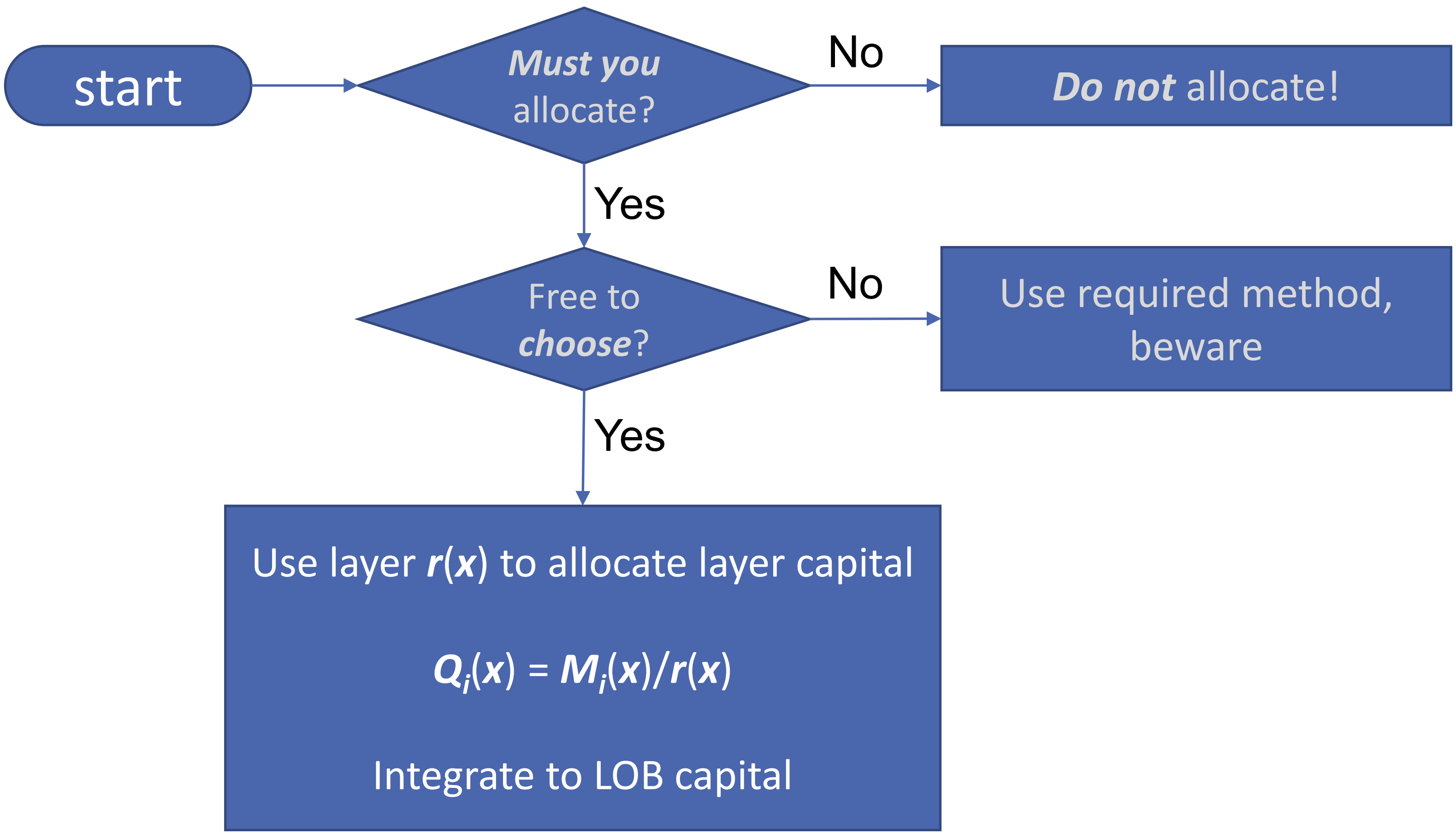
convex risk

Capital allocation recommendation flowchart



convex risk

Capital allocation recommendation flowchart



Law invariant assumption

A **law invariant** risk measure is function of the distribution of outcomes but does not distinguish by cause of loss

Therefore return can't vary by line within a layer

For a given **layer**, all LOBs must have the same ROE

$$r(x) = \frac{M(x)}{Q(x)} = r_i(x) = \frac{M_i(x)}{Q_i(x)}$$

Spectral risk measures are law invariant

Implied layer capital allocation by line

$$r(x) = \frac{M_i(x)}{Q_i(x)} \Rightarrow Q_i(x) = \frac{M_i(x)}{r(x)} = M_i\ (x) / \frac{g(S(x)) - S(x)}{1 - g(S(x))}$$

$$Q_i(x) = \frac{\beta_i(x) g(S(x)) - \alpha_i(x) S(x)}{g(S(x)) - S(x)} \left(1 - g(S(x))\right)$$

Capital allocation — Capital in layer

...how to allocate capital if you really must!

Margin and capital allocation can be negative!

- Margin can be negative if $\beta_i(x)$ sufficiently less than $\alpha_i(x)$

$$\frac{\beta_i(x)g\big(S(x)\big) - \alpha_i(x)S(x)}{g\big(S(x)\big) - S(x)}$$

- When is $\beta_i(x) < \alpha_i(x)$? For relatively thin tailed lines!

Cost of capital varies by amount of assets

- Total cost of capital = total required margin is a function of total assets

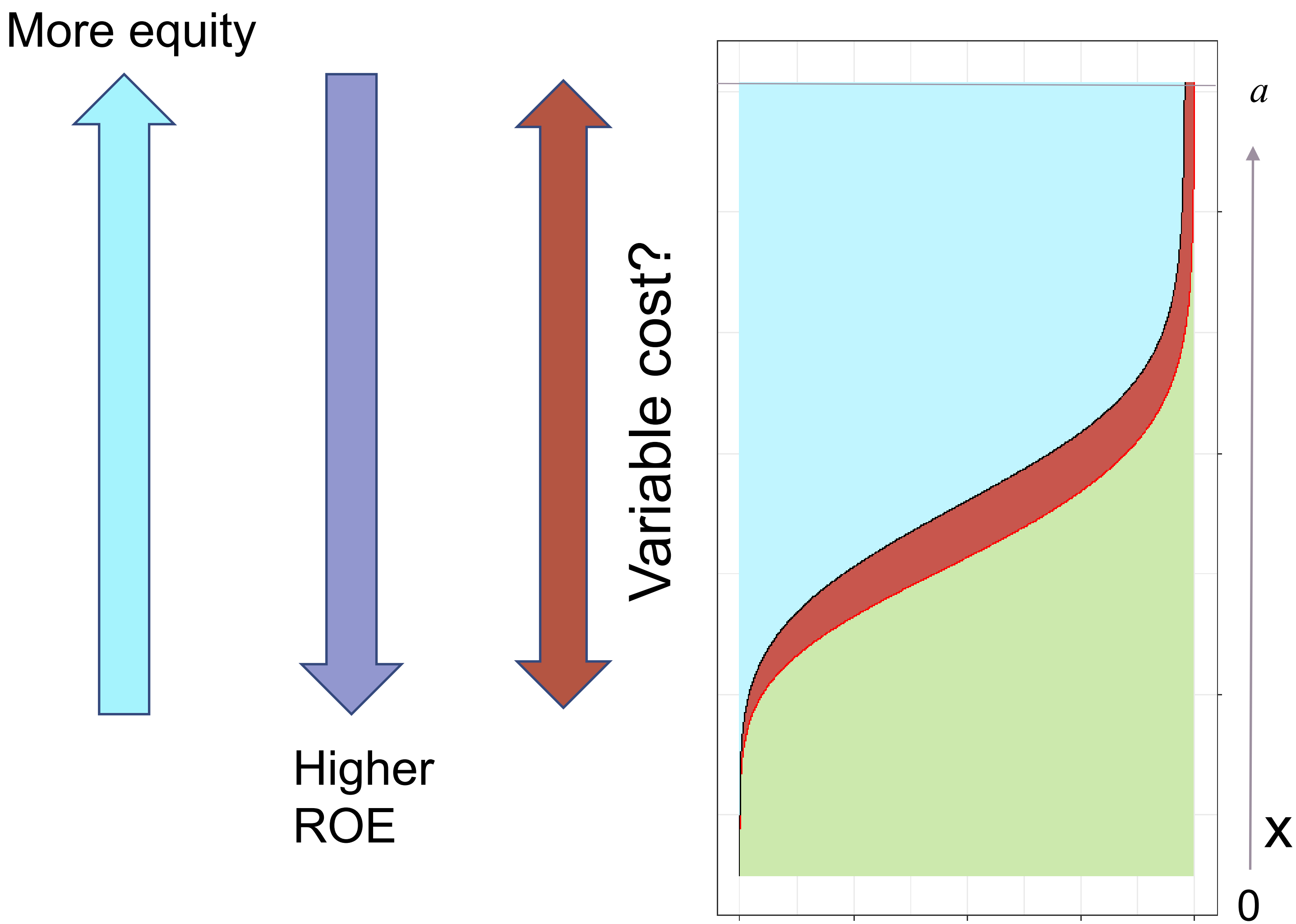
$$\bar{M}(a) = \int_0^{a} g(S(x)) - S(x)dx$$

- Total capital also varies with assets

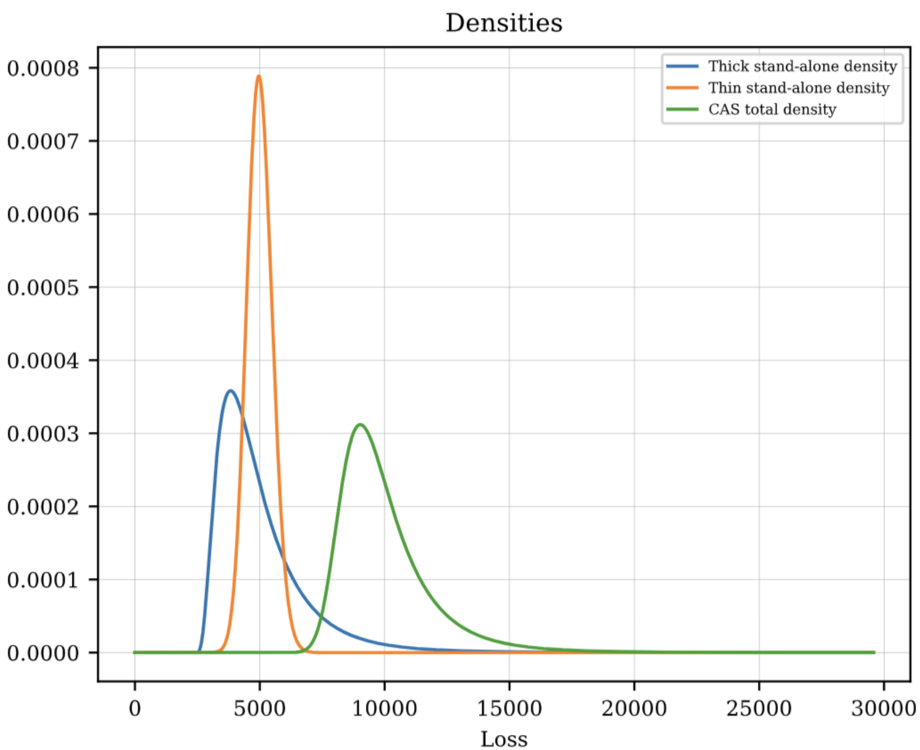
$$\bar{Q}(\mathrm{a}) = \int_0^{a} 1 - g(S(x))dx$$

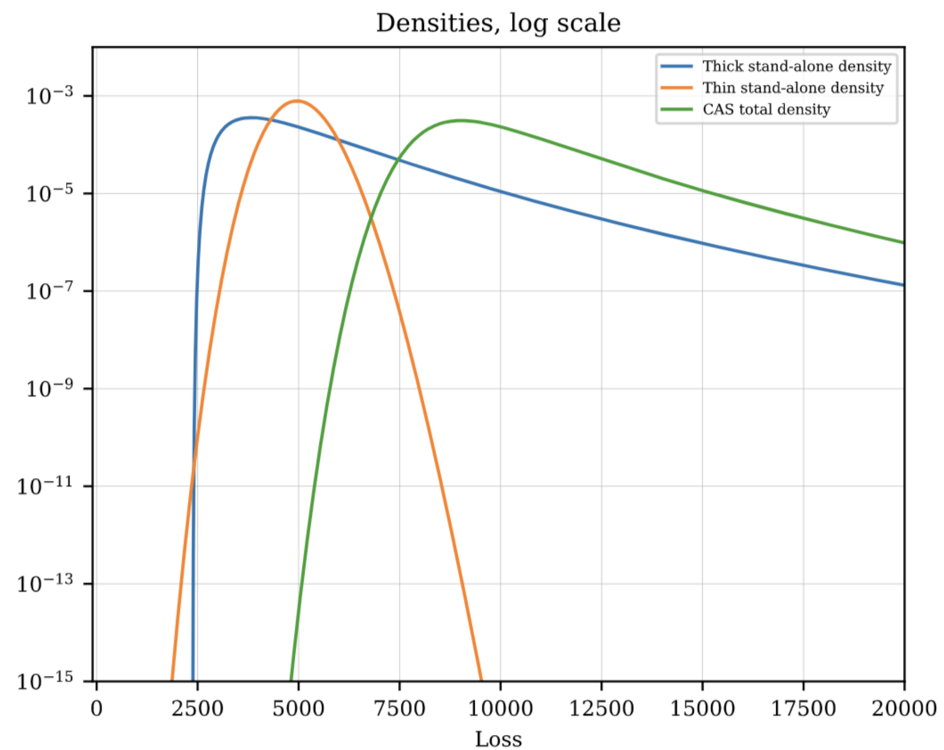
- Hence **cost of capital** (target ROE) **varies with assets**

Equity and margin vary by layer in complex manner



Example: Thick and Thin two-line model

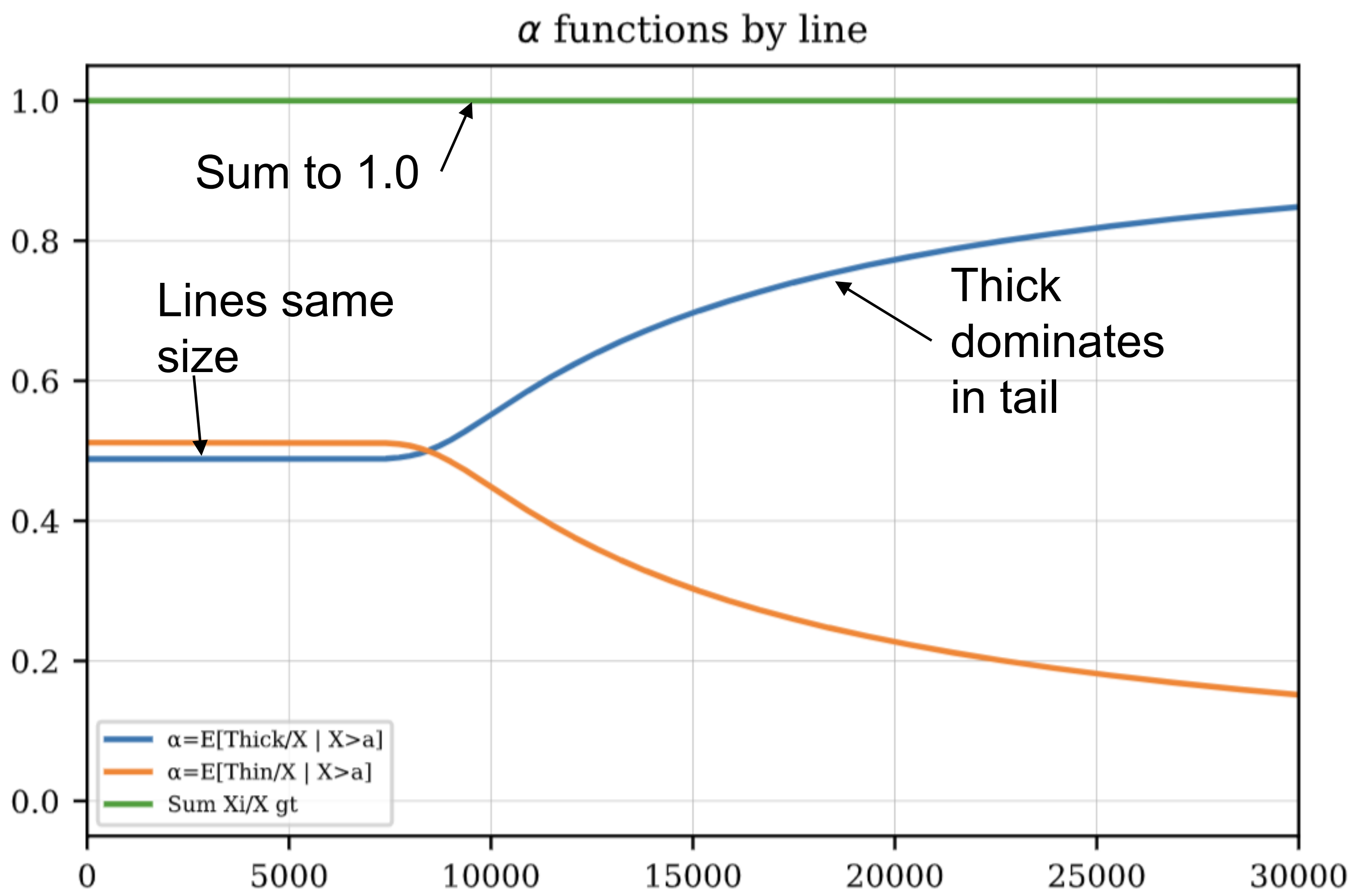




- Lines independent, convenience only
- Lines same size, each has EL = 5000
- Line CVs are 36% and 10%, overall CV = 18.9%
- Pricing: Wang distortion to 10% ROE at 20,000 assets, LR = 91.7%

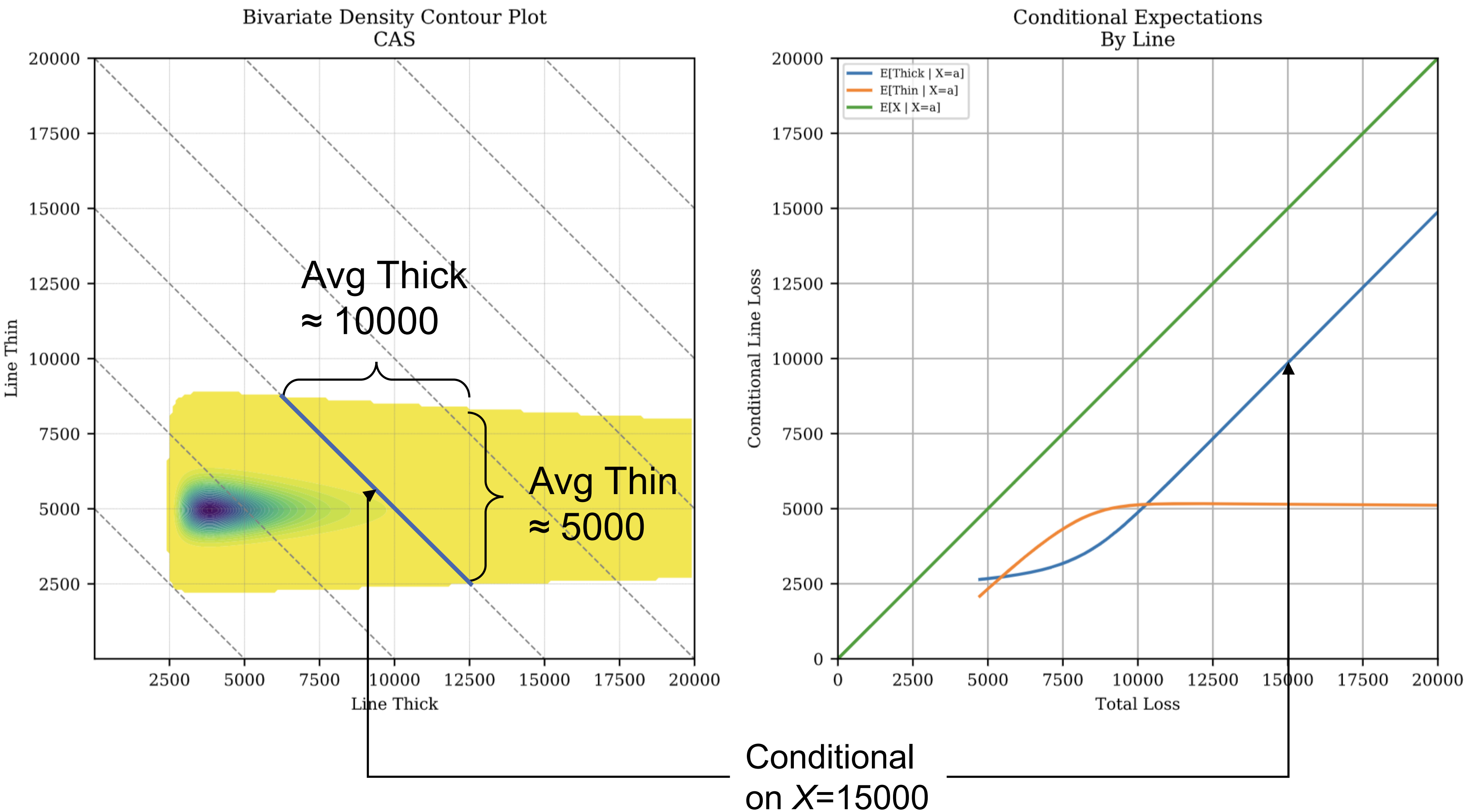
alpha function: calculates expected loss by line

- $\alpha_i(x)$ = E[X_i / X | X > x] as a function of x



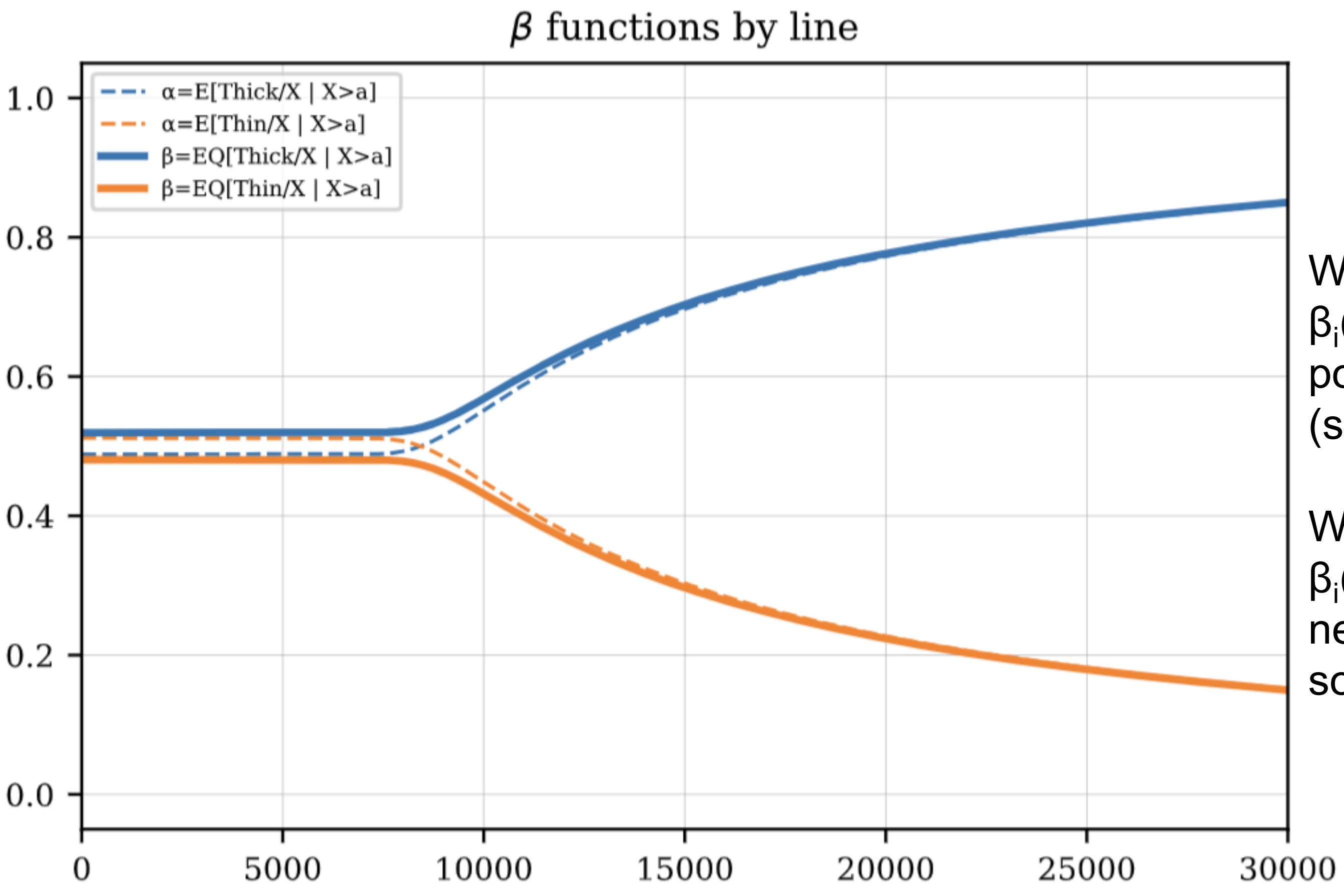
$$\alpha_i(x)S(x) = \int_x^\infty \frac{E[X_i \mid X = t]}{t} f_X(t)dt$$

E[X_i | X=x]: building block function for alpha and beta



beta function: calculates premium by line

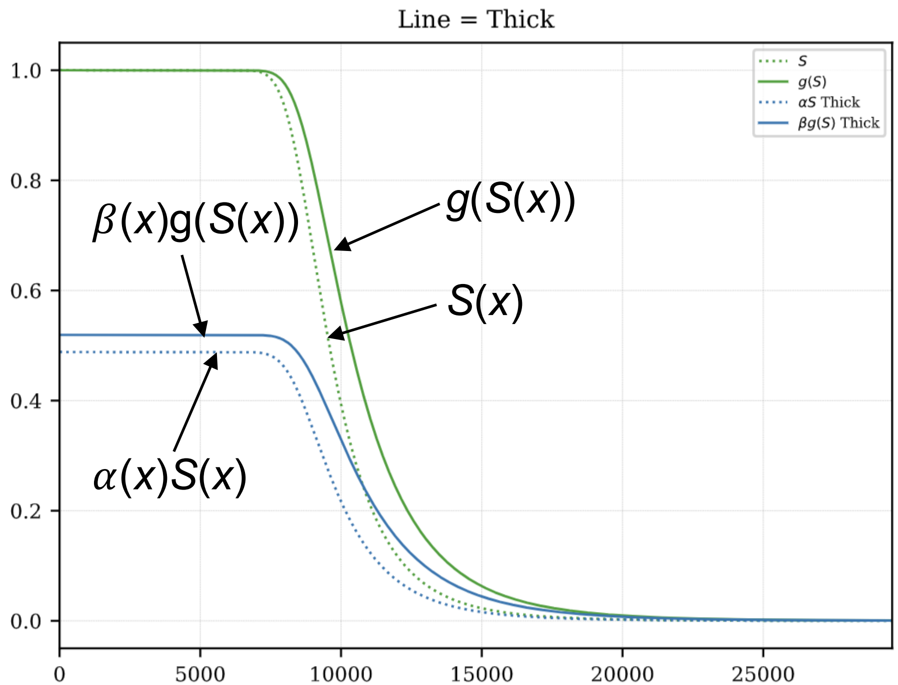
- $\beta_i(x) = E_g[X_i / X \mid X > x]$ (solid line) calculates premium
- Risk adjusted version of α, putting more weight on right tail

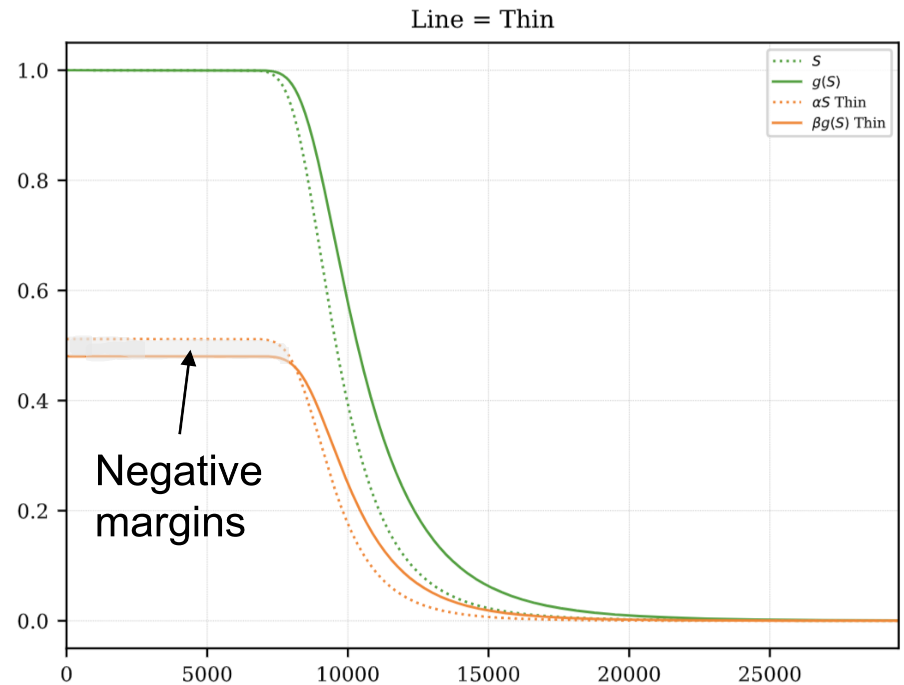


When $\alpha_i(x)$ **incr**eases $\beta_i(x)$ is **above** $\alpha_i(x)$, positive margins = **Thick** (solid above dashed)

When $\alpha_i(x)$ **decr**eases $\beta_i(x)$ is **below** $\alpha_i(x)$, negative margins for some layers = **Thin**

Margins by asset layer, by line and tail behavior





Thick... $\alpha_i(x)$ **increases**... $\beta_i(x)$ **above** $\alpha_i(x)$

$\beta_i(x)g(S(x))$ **above** $\alpha_i(x)S(x)$ since $g(S)>S$

Positive margins at all layers of capital

Thin... $\alpha_i(x)$ **decreases**... $\beta_i(x)$ **below** $\alpha_i(x)$

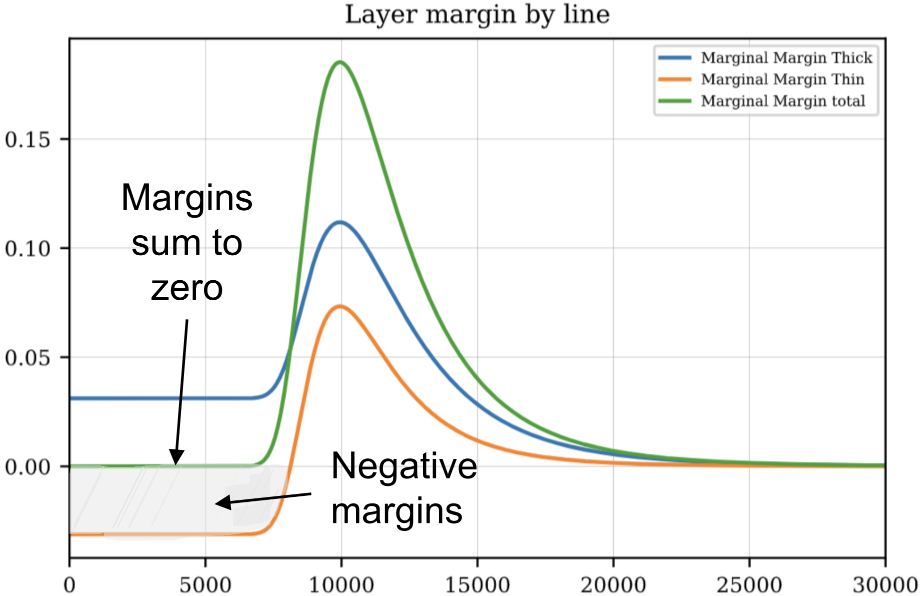
$\beta_i(x)g(S(x))$ may be **below** $\alpha_i(x)S(x)$

Possible negative margins for low layers, $g(1)=1$, and lower overall margin

Margin by asset layer, by line

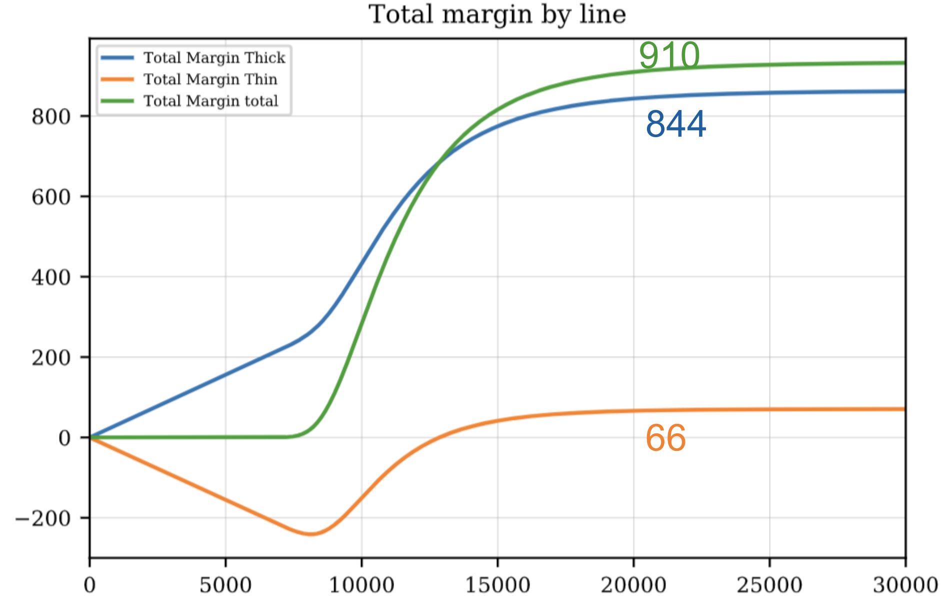
Layer Margin Density = Solid - Dashed

$\beta_i(x)g(S(x)) - \alpha_i(x)S(x)$



Cumulative Margin

Integral of margin density

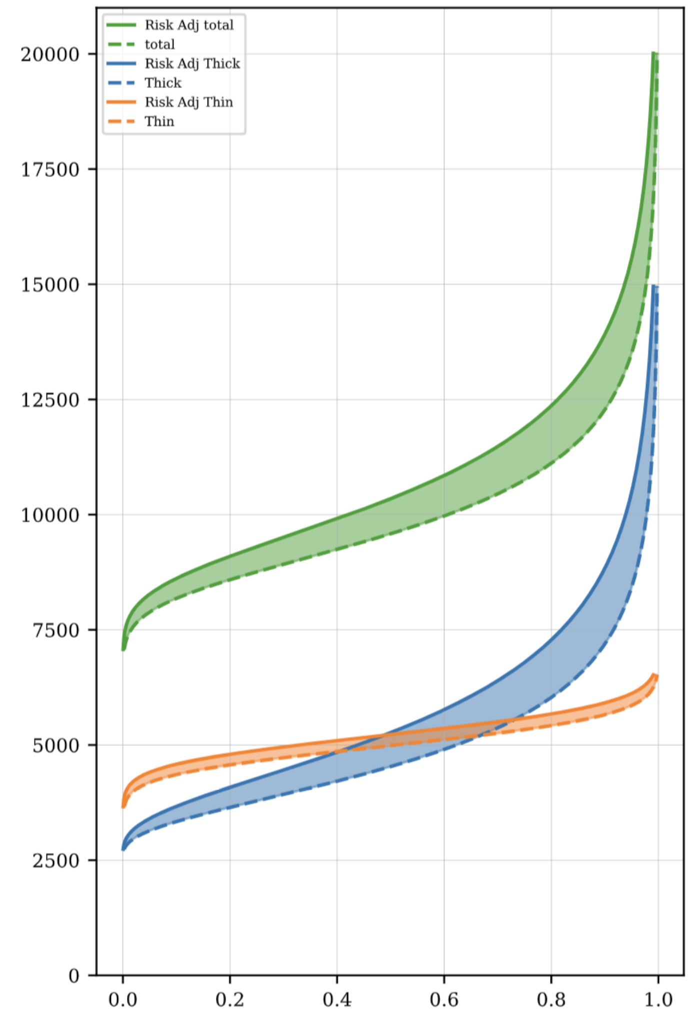


- Thick gains by pooling with thin in low layers
- Thick pays a positive margin to compensate thin for worse coverage
- Both lines benefit from better cover at high layers
- Both lines pay positive margin for incremental capital
- Above 13K both lines pay positive margin but thin line cost reduced by coverage impacts of pooling with Thick
- Thin 2.5% cost of capital and thick 13.1%; overall cost calibrated to 10.0% (see appendix for details)

Pricing summary: **not the tail wagging the dog**

- Thick line double whammy
  - Higher capital need
  - Consumes more high relative cost tail capital
- Pooling helps Thick, hurts Thin
- **Margin driven by body, not default**

- Total margins (shaded, right)
  - Combined **910** (right)
  - Thick within total 844 (prev. sld.)
  - Thin within total 66 (prev. sld.)
  - Thick stand-alone **872** (right)
  - Thin stand-alone **239** (right)



y axis shows losses to consistent probability threshold by line

convex risk

Pricing summary: **not** the tail wagging the dog

- Thick line double whammy
  - Higher capital need
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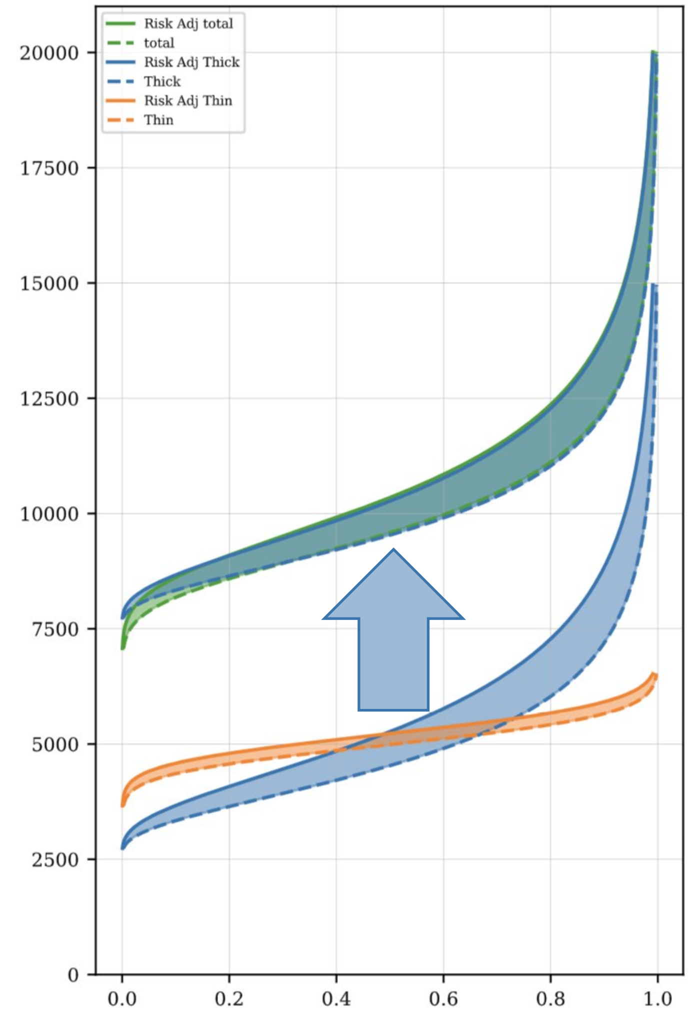
- Pooling helps Thick, hurts Thin
- **Margin driven by body, not default**

- Adding Thin line hardly changes shape or area of Thick line margin!
- Thick blue, translated up by 5000, expected loss for Thin, is almost the same total, green
- Adding thin ≈ adding constant loss



y axis shows losses to consistent probability threshold by line

Where to find thin-tailed business? Reserves!

Reformulate example as a two-period model

- Thick = current accident year
- Thin = reserves from prior year
- Paid losses from reserves at $t = 2$
- 1/1 effective date, no UPR
- Steady-state

- Policy year margins
  - Margin in premium **910**
  - Earned in year 1 **844**
  - Earned in year 2 **66**
- Amount earned in year 2 is the appropriate risk margin for reserves, c.f. IFRS, Solvency II
- Deferral matches earnings to delivery of insurance product and resolution of uncertainty
- With deferred income **allocating capital to reserves** makes more sense

Conclusions

- Premium based on **fair value to customers** of contractual cash flows and not **marginal cost to insurer**
  - Marginal cost view generally different, driven by regulatory capital standard
- Cost of capital varies by layer, line & amount of capital in a complex manner!
- No need to allocate capital to price or to evaluate pricing or performance!
- Link to videos: http://go.guycarp.com/cas2018
- Fully executable Python workbook with example: http://bit.ly/2TJs5id
- See forthcoming book / paper for details!

Appendix

convex risk

Audit statistics and pricing summary

	Thick	Thin	total
Mean	5000	5000	10000
CV	0.364418	0.101493	0.189144
Skew	2.40723	0.158277	2.1551
EmpSkew	2.4055	0.158277	2.15259
P99.0	11645	6240	16712
P99.5	13212	6384	18274
P99.99	24537	7067	29578
MeanErr	-4.73138e-07	-1.22125e-15	-4.88351e-07
CVErr	-2.41911e-05	2.28706e-14	-3.92099e-05

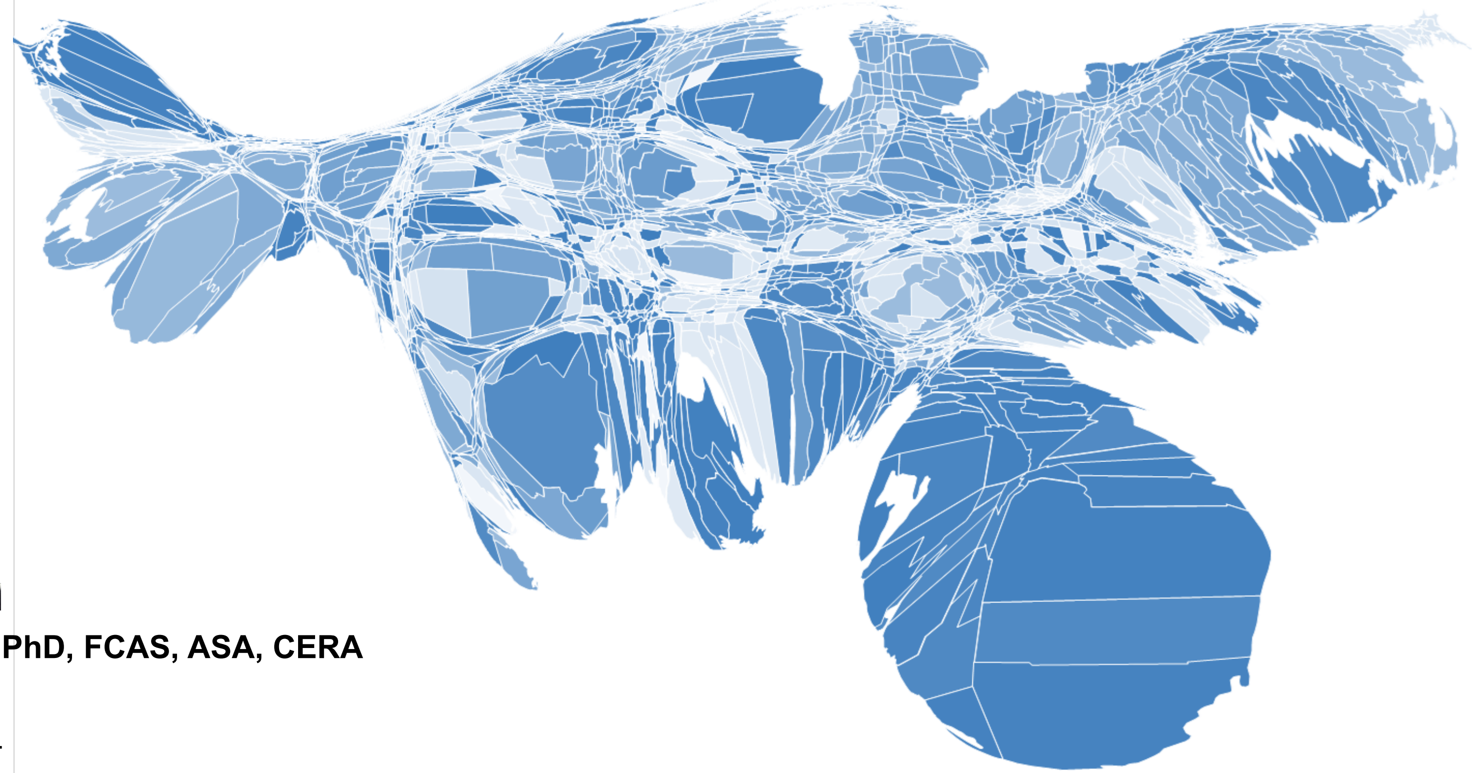
line	Thick	Thin	total
stat			
EPD	0.001107	0.00027622	0.00069138
Loss	4994.5	4998.6	9993.1
Loss Ratio	0.85552	0.98691	0.91656
Margin	843.44	66.28	909.72
Premium	5837.9	5064.9	10903
P/S Ratio	0.90642	1.9066	1.1985
Equity	6440.6	2656.6	9097.2
ROE	0.13096	0.02495	0.1

- Example produced using aggregate Python package https://github.com/mynl/aggregate
- pip install aggregate
- Aggregate portfolio specification:

```
port CAS
        agg Thick 5000 loss 100 x 0 sev lognorm 10 cv 20 mixed sig 0.35 0.6
        agg Thin  5000 loss 100 x 0 sev lognorm 10 cv 20 poisson
```

- Pricing results calibrated to 10% return at 20000 assets, p=0.997, using a Wang transform
- $P + Q$ = 10903 + 9097 = 20000
- $(P - L) / Q$ = (10903 – 9993) / 9097 = 0.1

Contact Information



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Graphic note: County size scaled to AAL estimates for hurricane, earthquake and severe weather using Gastner & Newman algorithm